International Journal of Information Technology, Research and Applications (IJITRA)
Vol.1, No.2, August 2022, pp. 16~22

Mari's algorithm to solve a fuzzy transportation problem by using Trapezoidal Fuzzy numbers

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Article Info

Article history:

Received June 06, 2022
Revised July 19, 2022
Accepted July 23, 2022

Keywords:

Transportation Problem
Fuzzy Logic
Optimization

ABSTRACT

In this article, the researcher studied Mari's Algorithm for solving Transportation problems. This research article discusses a modified solution methodology based on the existing method of solving a Transportation Problem [TP]. The author made an attempt to solve the Fuzzified Transportation Problem [FTP] using Modified Mari's Algorithm. Algorithm is a very simple and lucid method which passes many times the direct optimal solution. This method ensures that the optimal solution to the given FTP can be obtained with less time.

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1. INTRODUCTION

The word "transportation" is misleading because it seems to be limited to only transportation systems, but this is not the case. In reality, many resource utilisations issues that arise in manufacturing systems can be treated as transportation issues. Production scheduling, transportation scheduling, and so on are common examples. This problem was first presented by [4] Hitchcock (1941) and [5] Koopmans (1947), and [3] Dantzig (1947) solved it using the revised simplex method.

Hitchcock [4] developed the basic transportation problem in 1941, along with a constructive solution process, and Koopmans [5] addressed the problem in depth in 1949. Dantzig [3] reformulated the transportation problem as a linear programming problem and presented a solution method in 1951. Transportation has become a regular application for industrial organisations with multiple production facilities, warehouses, and distribution centres these days. It was necessary to divide the problem into two stages in order to achieve an optimal solution for transportation problems. In [6,9,11] the authors discussed the initial simple feasible solution (IBFS) was obtained in the first stage by using some of the available methods such as "North West Corner", "Matrix Minima", "Least Cost Method", "Row Minima", "Column Minima" and "Vogel's Approximation Method" among others. The MODI (Modified Distribution) approach was then used in the next and final stage to arrive at an optimal solution. The "Stepping Stone Method" was developed by Charnes and Cooper [2] for finding an optimal solution from IBFS. P. Pandian et al. [7], [8], and Sudhakar et al. [10] suggested two separate approaches for finding an optimal solution directly in 2010 and 2012, respectively. Mari's Algorithm was proposed by Mariappan, P., and Antony Raj M in [1] to solve the Transportation Problem.

Here's a much simpler process. For the Fuzzy Transportation Problem using Trapezoidal Fuzzy Numbers, Mari's Algorithm is used to find an optimal solution directly with fewer steps and using very simple computations. Using a flowchart method, the proposed method's step-by-step process is very clearly illustrated [Refer Flow chart – 3.1]. Example problems are used to demonstrate the proposed method's working methodology. This approach often yields a direct optimal solution. The significance is that this technique can easily skip the time-consuming MODI iterations.

2. DEFINITION OF TRAPEZOIDAL AND TRIANGULAR FUZZY NUMBERS [11]

If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w(x-a)}{b-a} \; if \; a \le x \le b \\ w \; if \; b \le x \le c \\ \dfrac{w(x-d)}{c-d} \; if \; c \le x \le d \\ 0 \; otherwise \end{cases}$$

If w = 1, then $\tilde{A}$ = (a, b, c, d; 1) is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or non-normal trapezoidal fuzzy number if 0 < w <1. The image of $\bar{A}$ = (a, b, c, d; w) is given by -$\tilde{A}$= ($-d$, $-c$, $-b$, $-a$; w).

In particular case if b = c, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A}$ = (a, b, d; w). The value of "b" corresponds with the mode or core and [a, d] with the support. If w = 1, then $\tilde{A}$ = (a, b, d) is a normalized triangular fuzzy number $\tilde{A}$ is a generalized or non-normal triangular fuzzy number if 0 < w < 1.

2.1 PROPERTIES OF TRAPEZOIDAL FUZZY NUMBER

Let $\tilde{A}$ = (a1, b1, c1, d1) and $\tilde{B}$ = (a2, b2, c2, d2) be two trapezoidal fuzzy numbers, then the fuzzy numbers addition and fuzzy numbers subtraction are defined as follows:
(i) Fuzzy numbers addition of $\tilde{A}$ and $\tilde{B}$ is denoted by $\tilde{A} \oplus \tilde{B}$ and is given by
$\tilde{A} \oplus \tilde{B}$ = (a1, b1, c1, d1) ⊕ (a2, b2, c2, d2) = (a1+ a2, b1+ b2, c1+ c2, d1+ d2)
(ii) Fuzzy numbers subtraction of $\tilde{A}$ and $\tilde{B}$ is denoted $\tilde{A} \ominus \tilde{B}$and is given by
$\tilde{A} \ominus \tilde{B}$ = (a1, b1, c1, d1) ⊖ (a2, b2, c2, d2) = (a1- d2, b1- c2, c1- b2, d1- a2)

2.2 NEW APPROACH FOR RANKING OF TRAPEZOIDAL FUZZY NUMBERS

Method of Magnitude

If $\tilde{a}$ = (a1, a2, a3, a4) is a trapezoidal fuzzy number, then the defuzzied value or the ordinary (crisp) number of, a is given below, a = $\frac{a_1+2a_2+2a_3+a_4}{6}$.We need the following definitions of ordering on the set of the fuzzy numbers based on the magnitude of a fuzzy number.

The magnitude of the trapezoidal fuzzy number, $\tilde{u}$ = (x_0−σ, x_0, y_0, y_0 + β) with parametric form $\tilde{u}$ = ($\bar{\bar{u}}$(r),$\bar{u}$(r)) where $\bar{\bar{u}}$(r) = x_0−σ+σr and $\bar{u}$(r) = y_0+β−βr is defined as

$$\text{Mag}(u) = \frac{1}{2}\left(\int_0^1 \bar{\bar{u}}(r) + \bar{u}(r) + x_0 + y_0\right) f(r)\, dr,$$

where the function f(r) is a non-negative and increasing function on [0,1], with f(0) = 0, f(1) = 1 and $\int_0^1$ f(r) dr = $\frac{1}{2}$.

Obviously function f(r) can be considered as a weighting function. In actual applications, function f(r) can be chosen according to the actual situation. The magnitude of a trapezoidal fuzzy number u which is defined by (1), synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. The resulting scalar value is used to rank the fuzzy numbers. In the other words Mag(u) is used to rank fuzzy numbers. The larger Mag(u) the larger fuzzy number.

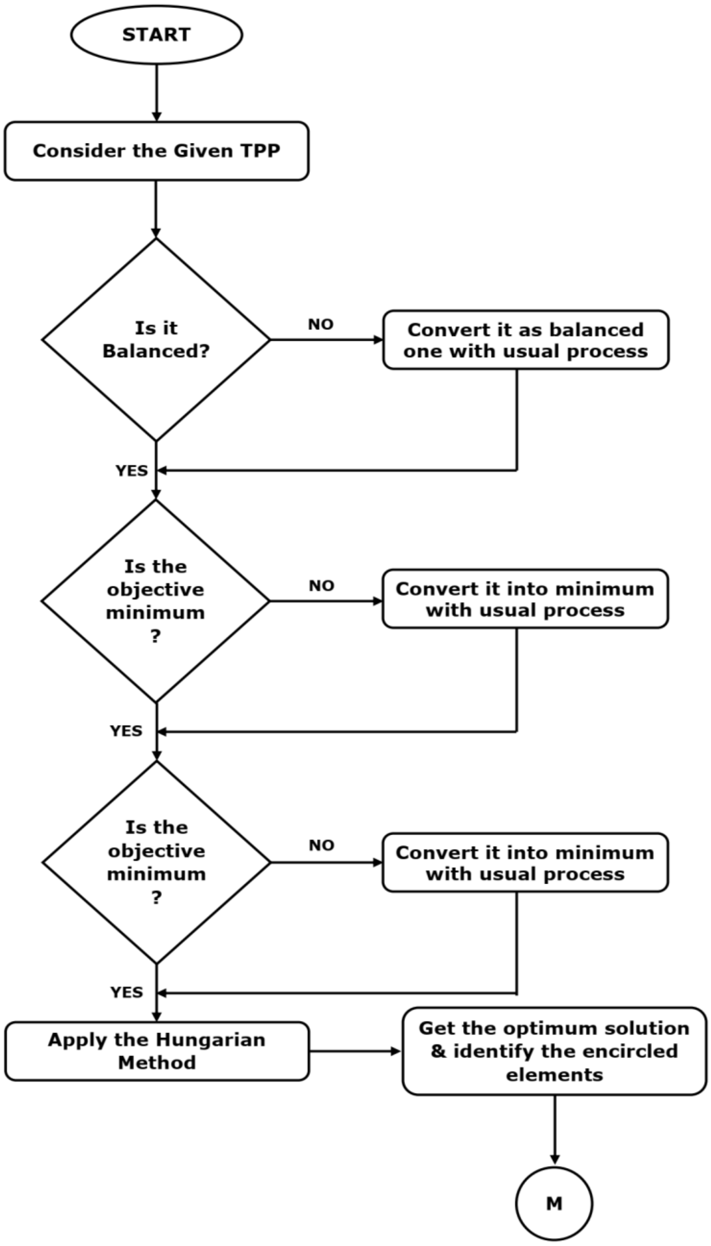
The magnitude of the trapezoidal fuzzy number $\tilde{u}$ = (a, b, c, d) is given by Mag($\bar{u}$) = $\frac{a+5b+5c+d}{12}$ or *Mag*(u) = $\frac{5}{12}(b+c) + \frac{1}{12}(a+d)$.

Let $\tilde{u}$ and $\tilde{v}$ be two trapezoidal fuzzy numbers. The ranking of $\tilde{u}$ and $\bar{u}$ by the Mag(u) on E, the set of trapezoidal fuzzy numbers is defined as follows:
(i) Mag($\tilde{u}$) > Mag($\tilde{v}$) if and only if $\tilde{u}$ > $\tilde{v}$;
(ii) Mag($\tilde{u}$) < Mag($\tilde{v}$) if and only if $\tilde{u}$ < $\tilde{v}$ and
(iii) Mag($\tilde{u}$)= Mag($\tilde{v}$) if and only if $\tilde{u}$ = $\tilde{v}$;

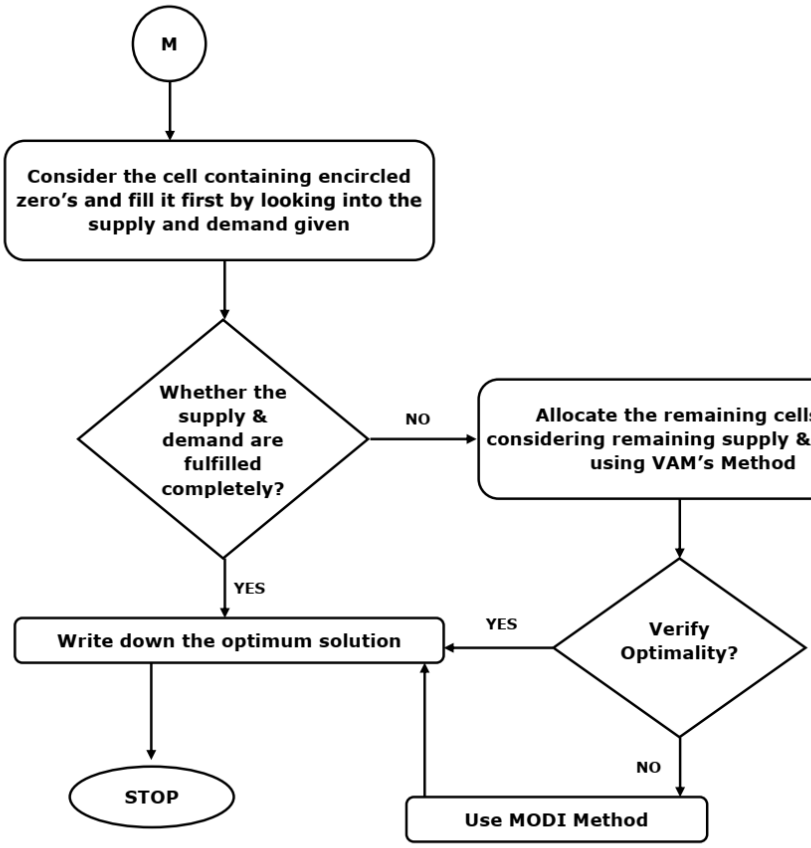
The ordering ≥ and ≤ between any two trapezoidal fuzzy numbers $\tilde{u}$ and $\tilde{v}$ are defined as follows:
(i) if $\tilde{u}$ ≥$\tilde{v}$;if and only if $\tilde{u}$ > $\tilde{v}$ or $\tilde{u}$ = $\tilde{v}$ and
(ii) if $\tilde{u}$ ≤$\tilde{v}$;if and only if $\tilde{u}$ < $\tilde{v}$ or $\tilde{u}$ = $\tilde{v}$.

The magnitude approach for ranking fuzzy numbers has some mathematical properties. It does not imply much computational effort and does not require a priori knowledge of the set of all alternatives. We also used comparative examples to illustrate the advantages of the proposed method.

3. PROBLEM FORMULATION



Flow Chart-3.1
Mari’s Algorithm [1]:



The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

Minimise = $\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$

Subject to $\sum_{j=1}^{q} x_{ij} = \widetilde{a_i}$, i=1,2,…….p

$\sum_{i=1}^{p} x_{ij} = \widetilde{b_j}$, j=1,2,…….q

$$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$$

Xij is a non- negative trapezoidal fuzzy number, Where,

p = total number of sources, Q = total number of destinations, a_i = the fuzzy availability of the product at ith source, b_j = the fuzzy demand of the product at jth destination, c_{ij} = the fuzzy transportation cost for unit quantity of the product from ith source to jth destination, x_{ij} = the fuzzy quantity of the product that should be transported from ith source to jth destination to minimize the total fuzzy transportation cost. $\sum_{i=1}^{p} a_i = 1$, total fuzzy availability of the product, $\sum_{j=1}^{q} b_j = 1$,total fuzzy demand of the product, $\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$= 1 total fuzzy transportation cost.

If $\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem. Consider transportation with m fuzzy origin s (rows) and n fuzzy destinations (Columns) Let $C_{ij} = [C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}]$ be the cost of transporting one unit of the product from ith fuzzy origin to jth fuzzy destination $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$ be the quantity of commodity available at fuzzy origin i $b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}]$ be the quantity of commodity requirement at fuzzy destination j. $X_{ij} = [X_{ij}^1, X_{ij}^2, X_{ij}^3]$ is quantity transported from ith fuzzy origin to jth fuzzy destination.

4. NUMERICAL EXAMPLE

Consider the fuzzy transportation problem[12].

The following table shows all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs) from each warehouse to each market. Here cost value, supplies and demands are trapezoidal fuzzy numbers. Here FOi and FDi are Fuzzy Supply and Fuzzy Demand. The given problem is balanced transportation problem. There exists fuzzy initial basic feasible solution.

Example: 4.1

Let us solve the balanced fuzzy transportation problem for maximizing the profit,

	FD 1	FD2	FD3	FD4	Fuzzy Available
F01	(1, 2, 3, 4)	(1, 3, 4, 6)	(9, 11, 12, 14)	(5, 7, 8, 11)	(1, 6, 7, 12)
F02	(0, 1, 2, 4)	(-1, 0, 1, 2)	(5, 6, 7, 8)	(0, 1, 2, 3)	(0, 1, 2, 3)
F03	(3, 5, 6, 8)	(5, 7, 8, 10)	(12, 15, 16, 19)	(7, 9, 10, 12)	(5, 10, 12, 15)
Fuzzy requirement	(5, 7, 8, 10)	(-1, 5, 6, 10)	(1, 3, 4, 6)	(1, 2, 3, 4)	(6, 17, 21, 30)

By using method of magnitude for defuzzyfing the trapezoidal fuzzy numbers,

Mag(u) = $\frac{5}{12}(b+c)+\frac{1}{12}(a+d)$.

R(1,2,3,4) = 2.5, R(1,3,4,6) =3.5 , R(9,11,12,14) =11.5,
R(5,7,8,11) = 7.5 R(1,6,7,12) = 6.5
R(0,1,2,4)= 1.5, R (-1,0,1,2)= 0.5, R (5,6,7,8)= 6.5,
R(0,1,2,3)= 1.5, R (0,1,2,3)= 1.5
R(3,5,6,8)=5.5,R(5,8,9,12)=8.5,R (12,15,16,19)= 16.5,
R(7,9,10,12)=9.5, R(5,10,12,15)=10.8
R(5,7,8,10)= 7.5 ,R (-1,5,6,10)=5.3 ,R (1,3,4,6) =3.5,
R (1,2,3,4)= 2.5,R (6,17,21,30)=18.8

We get the following

	FD1	FD2	FD3	FD4	Fuzzy available
F01	2.5	3.5	11.5	7.5	6.5
F02	1.5	0.5	6.5	1.5	1.5
F03	5.5	8.5	15.5	9.5	10.8
Fuzzy Requirement	7.5	5.3	3.5	2.5	18.8

By using the above new algorithm for solving the transportation problem we get the following allocations.

2.5	3.5	11.5	7.5		0	1	9	5		0	1	8	4		0	**0**	7	3
1.5	0.5	6.5	1.5		1	0	6	1		1	0	5	0		2	0	5	**0**
5.5	8.5	15.5	9.5		0	3	10	4		0	3	9	3		**0**	2	8	2
0.0	0.0	0.0	0.0		0	0	0	0		1	1	0	0		2	1	**0**	0

2.5	$3.5^{\mathbf{5.3}}$	$11.5^{\mathbf{1.2}}$	7.5
1.5	0.5	6.5	$1.5^{\mathbf{1.5}}$
$5.5^{\mathbf{7.5}}$	8.5	$15.5^{\mathbf{2.3}}$	$9.5^{\mathbf{1}}$
0.0	0.0	$0.0^{\mathbf{0}}$	0.0

Optimal solution

The transportation cost according to the Modified Mari's method is:

Total Cost = (3.5x5.5)+(11.5 x1.2)+(1.5 x1.5)+(5.5 x7.5)+(15.5 x2.3)+(9.5 x 1) = 121

The result obtained from Modified Mari's method with magnitude of ranking is compared with the exisisting results namely (i) Fuzzy Zero Method (ii Fuzzy VAM and,(iii) Fuzzy MODI Method.

Comparison Table is given bellow.

Example reference	Fuzzy zero method	Fuzzy VAM	Fuzzy MODI Method	Modified Mari's Method
1	132.17	122.5	121	**121**

Example: 4.2

	FD 1	FD2	FD3	FD4	Fuzzy Available
F01	(-4, 0, 4, 16)	(-4, 0, 4, 16)	(-4, 0, 4, 16)	(-2, 0, 2, 8)	(0, 4, 8, 12)
F02	(8, 16, 24, 32)	(8, 14, 18, 24)	(4, 8, 12, 16)	(2, 6, 10, 14)	(4, 8, 18, 26)
F03	(4, 8, 18, 26)	(0, 12, 16, 20)	(0, 12, 16, 20)	(8, 14, 18, 24)	(4, 8, 12, 16)
Fuzzy requirement	(2, 6, 10, 14)	(2, 2, 8, 12)	(2, 6, 10, 14)	(2, 6, 10, 14)	(8, 20, 38, 54)

We get the following

	FD1	FD2	FD3	FD4	Fuzzy available
F01	2.6	2.6	2.6	1.3	6
F02	20	16	10	8	13.3
F03	13.3	13.3	13.3	16	10
Fuzzy Requirement	8	5.3	8	8	29.3

2.6	2.6	2.6	1.3
20	16	10	8
13.3	13.3	13.3	16
0.0	0.0	0.0	0.0

1.3	1.3	1.3	0
12	8	2	0
0	0	0	2.7
0.0	0.0	0.0	0.0

0	0	0	0
10.7	6.7	0.7	0
0	0	0	4
0.0	0.0	0.0	13

2.6^{6}	2.6	2.6	1.3
20	16	$10^{5.3}$	8^{8}
13.3^{2}	$13.3^{5.3}$	$13.3^{2.7}$	16
0.0	0.0	0.0	0.0

The transportation cost according to the Modified Mari's method is:
Total Cost = (2.6x6) +(8x8)+(10x5.3)+(13.3x2.7)+(13.3x5.3)+(13.3x2)+ =265.6
The result obtained from Modified Mari's method with magnitude of ranking is compared with the existing results namely (i) Fuzzy Zero Method (ii) Fuzzy VAM and,(iii) fuzzy MODI Method.
Comparison Table is given bellow.

Example reference	Fuzzy Zero method	Fuzzy VAM	Fuzzy MODI	Modified Mari's Method
2	270.2	272	267.2	256.6

5. CONCLUSION

As a result, it is self-evident that Mari's Algorithm provides an optimal solution for the Fuzzy Transportation Problem. This approach uses only the simple Hungarian algorithm and, on occasion, Vogel's Algorithm. It is sometimes necessary to enlist the assistance of MODI in order to progress further. Since the solution takes less time and is simple to understand, it can be considered a good alternative to the conventional methods for resolving a Fuzzy Transportation Problem.

REFERENCES

[1] Antony Raj, M. (2018). Matematical Modelling in Operations Research for an Effective Managerial Decision Making, Ph.D. Thesis, Bharathidasan University, Trichy.

[2] Charnes, A., Henderson, A., & Cooper, W.W. (1955). An introduction to linear programming. John Wiley & Sons.

[3] Dantzig, G. B. (1965). Linear programming and extensions (Vol. 48). Princeton university press.

[4] Hitchcock, F. L. (1941). The distribution of a product from several sources tp numerous localities. Journal of mathematics and physics, 20(1-4), 224-230

[5] Koopmans, T. C. (1949). Optimum utilization of the transportation system. Econometrica: Journal of Econometric Society, 136-146.

[6] Mariappan, P, Operations Research: An Introduction, Pearson, New Delhi, ISBN: 978-81-317-9934-5.

[7] Pandian, P., & Natarajan, G. (2010). A new method for finding an optimal solution for transportation problems. International J. of Math. Sci. and Engg. Appls, 4, 59-65.

[8] Pandian, P., & Natarajan, G. (2010). A New Algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Applied mathematical sciences, 4(2), 79-90.

[9] Sharma. J.K. (2005). Operations Research-Theory and applications, Macmillan India (LTD), New Delhi.

[10] Sudhakar, V. J., Arunsankar, N., & Karpagam, T. (2012). A New approach for finding an Optimal solution for transportation problems. European journal of scienctific research, 68(2), 254-257.
[11] Taha. H.A. (2004). Operation Research – Introduction, Prentice Hall of India (PVT), New Delhi.
[12] Vimala, S., & Prabha, S. K. (2016). Fuzzy transportation problemthrough monalisha's approximation method. Journal of Advances in Mathematics and Computer Science, 1-11.

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