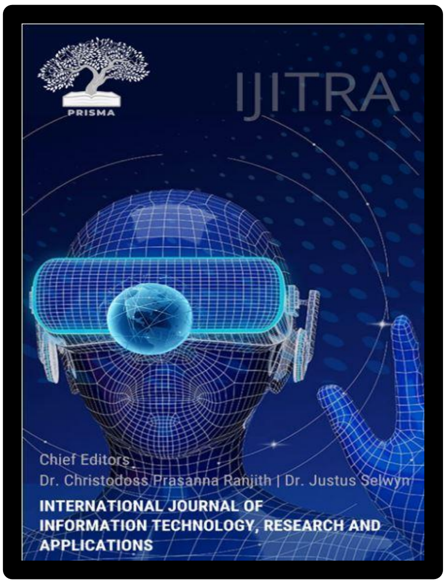

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On Degree Product Graphs

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ABSTRACT

In this paper we introduce a new concept called degree product graphs of G_1 and G_2 denoted by $G_1 * G_2$. We apply this concept to some of the regular and non-regular degree product graphs. We have obtained some general results as well.

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1. INTRODUCTION

In this paper all graphs considered are simple, connected and finite. Let $G = (V(G), E(G))$. In this paper we introduce a new concept called degree product graphs of G_1 and G_2 denoted by $G_1 *$ has the vertex set $V(G_1 * G_2) = V(G_1) \times V(G_2)$ and two vertices of $G_1 * G_2$ are said to be adjacent if (u_1, v_1) and (u_2, v_2) have the same order pair of degree.

Be a connected graph of order n. For any vertex $u \in V(G)$, the degree of the vertex is the total number of edges incident with them is denoted by $d_G(u)$. Let K_n denote a complete on n vertices. Notation and definitions which are not given here can be found in (Buckley & Harary, 1990). or (Chartrand & Zhang 2004).

So far different types of product graphs were introduced like Cartesian product, tensor product, wreath product of the graphs. The Cartesian product of the graphs G_1 and G_2 denoted by $G_1 \boxdot G_2$, has the vertex set $V(G_1 \boxdot G_2) = V(G_1) \times V(G_2)$ and $(u, x)(v, y)$ is an edge of $G_1 \boxdot G_2$ if $u = v$ and $xy \in E(G_2)$ or $uv \in E(G_1)$ and $x = y$. The wreath product of the graphs G_1 and G_2 denoted by $G_1 \circ G_2$ is the graph with vertex set $V(G_1) \times V(G_2)$ and $(u, x)(v, y)$ is an edge, whenever $(i) uv \in E(G_1)$ (or) $(ii) u = v \; and \; xy \in E(G_2)$.

The tensor product of G_1 and G_2 is denoted by $G_1 \otimes G_2$ is the graph with vertex set $V(G_1) \times V(G_2)$ and two vertices $u = (u_1, v_1), v = (u_2, v_2)$ are said to be adjacent if u_1 is adjacent to u_2 in G_1 and v_1 is adjacent to v_2 in G_2.

The topological index is a numerical quality related to a graph that is invariant under graph automorphisms. A topological index related to distance is called a “distance based topological index”. The

Wiener index $W(G)$ is the first distance based topological index defined as $W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v) = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$ with the summation going over all pairs of vertices of G. The topological indices and graph invariants based on distance between vertices of a graph are widely used for characterizing molecular graphs, establishing relationships between structure and properties of molecules, predicting biological activity of chemical compounds, and making their chemical applications (Trinajstic 1983). The Wiener index is one of the more used topological indices with high correlation with many physical and chemical indices of molecular compounds.

There are some topological indices based on degrees such as the first and second Zagreb indices of molecular graphs. The first and second kinds of Zagreb indices were first introduced in (Gutman et.al 1975). (see also Gutman &Trinajstic (1972). It is reported that these indices are useful in the study of anti-inflammatory activities of certain chemical instances, and in other practical aspects.

The first Zagreb index $M_1(G)$ and second Zagreb index $M_2(G)$ of a graph G are defined as

$$M_1(G) = \sum_{u,v\in E(G)} [d_G(u) + d_G(v)] = \sum_{v\in V(G)} d_G^2(v)$$

$$M_2(G) = \sum_{uv\in EG)} [d_G(u)d_G(v)]$$

The degree distance was introduced by (Dobrynin & Kochetova 1994). and (Gutman 1994) as a weighted version of the Wiener index. The degree distance of G, denoted by $DD(G)$, is defined as

$$DD(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v)[d_G(u) + d_G(v)] = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)[d_G(u) + d_G(v)]$$

With the summation going all pair of vertices of G. The degree distance is also known as Schultz index in chemical literature; see (Schultz 1989). In (Chen & Guo 2010). It has been demonstrated that the Wiener index and the degree distance are closely and mutually related for certain classes of molecular graphs.

The definitions of the terms eccentricity, radius, diameter, girth and center of a graph G is as follows:

Let G be a graph and v be a vertex of G.

The *eccentricity* of the vertex v is the maximum distance from v to any vertex. That is, $e(v) = max\{d(v,w): w\ in\ V(G)\}$.

The *radius* of G is the minimum eccentricity among the vertices of G.

Therefore, $radius(G) = min\{e(v): v\ in\ V(G)\}$.

The *diameter* of G is the maximum eccentricity among the vertices of G.

Thus, $diameter(G) = max\{e(v): v\ in\ V(G)\}$.

The *girth* of G is the length of a shortest cycle in G.

The *center* of G is the set of vertices of eccentricity equal to the radius.

Hence, $center(G) = \{v\ in\ V(G): e(v) = radius(G)\}$.

the **degree** (or **valency**) of a vertex of a graph is the number of edges that are incident to the vertex, and in a multigraph, loops are counted twice. The degree of a vertex vv is denoted by $deg\ deg\ (v)$. or . The **maximum degree** of a graph G, denoted by $\triangle(G)$,and the **minimum degree** of a graph, denoted by $\delta\delta(G)$.

Let us discuss some of the basic and important definitions which we are dealing in this paper.

Complete Graph

A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

Regular Graph

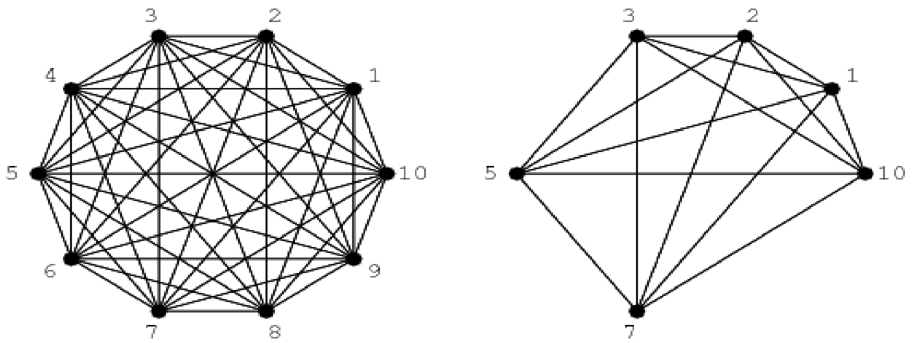
A regular graph is a graph where each vertex has the same number of neighbors (i.e.) every vertex has the same degree on valency.

Cyclic Graph

A cyclic graph is a graph containing at least one graph cycle.

Induced subgraph

A vertex-induced subgraph (sometimes simply called an "induced subgraph") is a subset of the vertices of a graph G together with any edges whose endpoints are both in this subset.



The figure above illustrates the subgraph induced on the complete graph K_{10} by the vertex subset $\{1,2,3,5,7,10\}$.

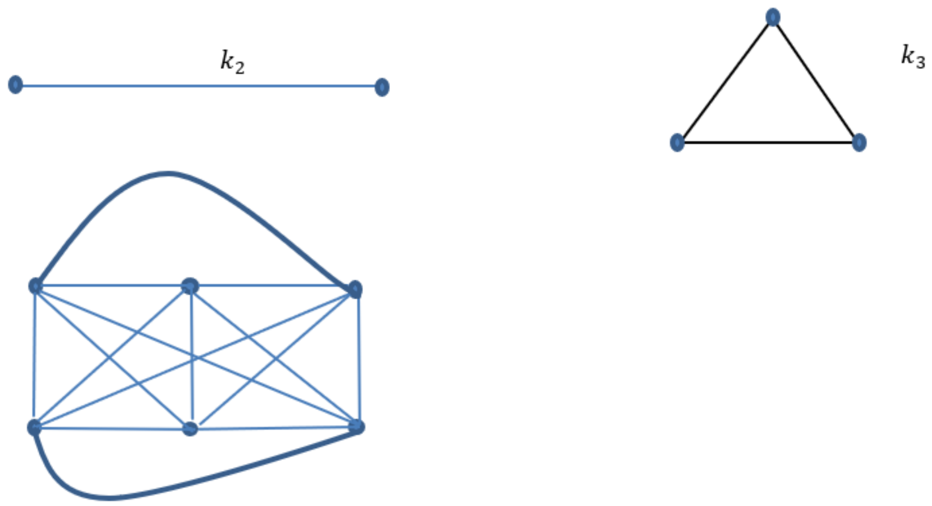
Connected Graph

A graph which is said to be connected if there is a path from any point to any other point in the graph. A graph which is not connected is said to be disconnected.

Theorem 1: Degree product of two complete graphs is a complete graph

Proof: Two graphs G_1 and G_2 are complete graphs with n and m vertices respectively. G_1 has $\frac{n(n-1)}{2}$ edges and G_2 has $\frac{m(m-1)}{2}$ edges. Degree product of G_1 and G_2 is denoted by $G_1 * G_2$ having $n \times m$ vertices say 'k'' vertices. We have to prove $G_1 * G_2$ is having $\frac{k(k-1)}{2}$ edges.

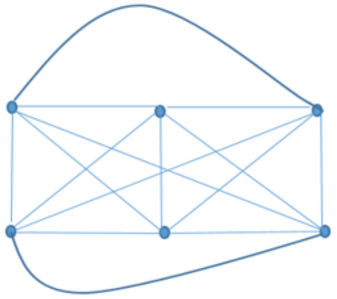
By the definition of complete graph, the degree of all vertices are same. so it is enough to prove that $G_1 * G_2$ of all vertices having degree $k-1$, where $k = n \times m$. Degree of all vertices in G_1 is $n-1$ and degree of all vertices in G_2 is $m-1$. Hence by definition of degree product graph, degree of all vertices in $G_1 * G_2$ is $(n-1, m-1)$. If the vertices having same degree then there exist an edge between them. Hence all vertices are connected between direct edge, hence $G_1 * G_2$ of all vertices having degree $k-1$, where $k = n \times m$. Hence $G_1 * G_2$ is having $\frac{k(k-1)}{2}$ edges



$k_2 \circledast k_3 = k_6$

Theorem 2: Degree product of two regular graphs is a complete graph

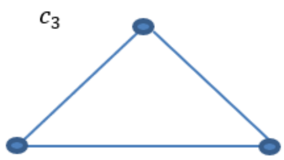
Proof: By theorem 1.

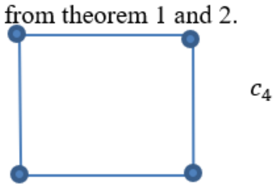


$c_3 \circledast p_2 = k_6$

Theorem 3: Degree product of two cyclic graphs is a complete graph

Proof: It is easy to prove from theorem 1 and 2.





$c_3 \circledast c_4 = k_{12}$

Theorem 4: If k_n and k_m be the complete graphs of n and m vertices respectively. Then degree product of k_n and k_m has $\frac{n^2m^2-nm}{2}$ edges.

Proof:

Let $k_n$be the completer graph then it has $\frac{n(n-1)}{2}$ edges.

Let $k_m$be the completer graph then it has $\frac{m(m-1)}{2}$ edges.

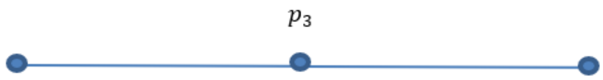
To prove that, $k_n * k_m$ has $\frac{n^2m^2-nm}{2}$ edges, it is enough to prove that degree product of $k_n * k_m$ is $k_{n\times m}$ complete graph. By the definition, $k_n * k_m$has $n \times m$ vertices. For regular graph all vertices having same degree, hence by the definition, $k_n * k_m$ has same order pair of degree, all vertices are adjacent.

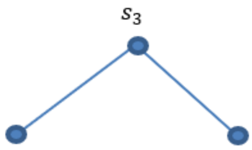
Hence proved the theorem.

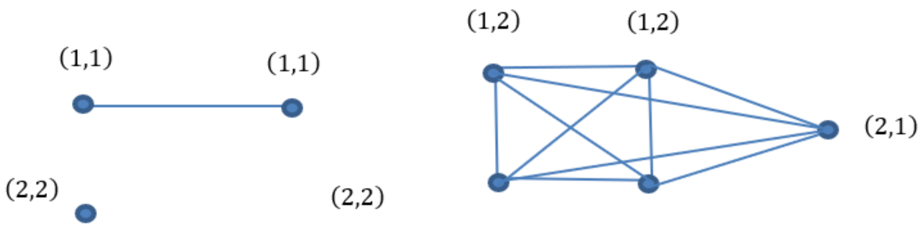
Theorem 5: Degree product of any two non-regular graphs are disconnected graphs.

Proof:

By the definition of degree product graph. There exist an edge between same degree pair of vertices in the graph. At least one edge between 1 pair of vertices having same degree. But there are edges between different pair of edges. So the degree product graphs are disconnected.







Theorem 6: Induced subgraph of degree product of any two non-regular graphs are complete.

Theorem 7: Degree product of two completer graphs having radius and diameter is equal to 1.

Theorem 8: Degree product of n complete graph is also a complete graph. (from theorem 1)

Theorem 9: Degree product of n regular graph is also a complete graph. (from theorem 2)

Theorem 10: Degree product of n cyclic graph is also a complete graph. (from theorem 3)

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