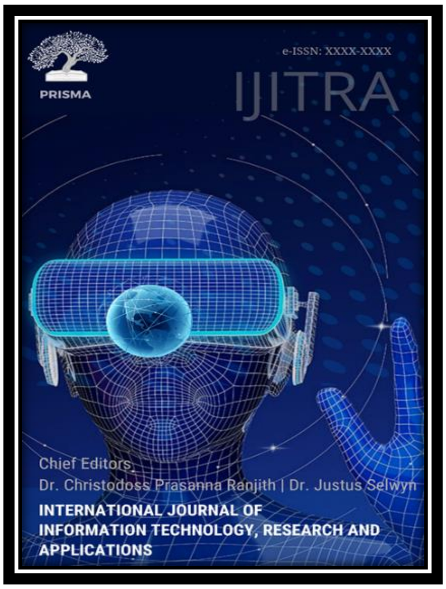

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Kenmotsu Manifolds Admitting a Non-Symmetric Non-Metric Connection

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ABSTRACT

The aim of the present paper is to study the properties of Kenmotsu manifolds equipped with a non-symmetric non-metric connection. We also establish some curvature properties of Kenmotsu manifolds. It is proved that a Kenmotsu manifold endowed with a non-symmetric non-metric is irregular.

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1. LITERATURE REVIEW

A contact metric manifold is capable to resolve many issues of sciences, engineering and medical sciences, and hence it is attracting the researchers to work in this area. Boothby & Wang (1958) started the study of a differentiable manifold with contact and almost contact metric structures. Kenmotsu (1971) introduced a class of almost contact metric manifold and named as Kenmotsu manifold. Since then, the properties of Kenmotsu manifolds have studied by several authors such as De & Pathak (2004), Sinha & Srivastava (1991), Jun, De & Pathak (2005), De, Yildiz and Yaliniz (2008), Chaubey et al. (2010, 2012, 2015, 2018), De (2008), Cihan (2006) and many others.

Let M be a Riemannian manifold associated with the Riemannian metric g. A linear connection $\tilde{\nabla}$ on M is said to be symmetric if the torsion tensor $\tilde{T}$ of $\tilde{\nabla}$ vanishes, otherwise it is non-symmetric. If the torsion tensor $\tilde{T}$ assumes the form $\tilde{T}(X,Y) = \pi(Y)X - \pi(X)Y$ for all vector fields X and Y on M, then then linear connection $\tilde{\nabla}$ is called semi-symmetric connection (Friedmann, & J. A. Schouten, 1924). Moreover, if $\tilde{\nabla}g =$

0 on M then $\tilde{\nabla}$ is said to be metric, otherwise non-metric (Agashe & Chafle, 1992). Chaubey & Ojha (2008) defined and studied the properties of non-symmetric non-metric connection on almost contact metric manifolds. The geometrical properties of the same connection have been studied by several authors. We refer (Chaubey & Kumar (2010); Chaubey & De (2019); Chaubey, & Yildiz (2019); Chaubey et al. (2019)) and their references.

Above studies motivate us to study the properties of Kenmotsu manifold equipped with a non-symmetric non-metric connection.

2. PRELIMINARIES:

A smooth manifold M of dimension (2n+1) is said to be an almost contact metric manifold if it admits a (1, 1) tensor field ϕ, (1, 0) type vector field ξ, (0, 1) type vector field η and a compatible metric g of type (0, 2) satisfies

$$\phi^2 = I + \eta \otimes \xi, \ \eta(\xi) = 1 \quad \text{and} \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \qquad (2.1)$$

for all X and Y on M (Blair ,1976).

Additionally, if M satisfies

$$\nabla_X \xi = X - \eta(X)\xi \Leftrightarrow (\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y) \qquad (2.2)$$

for all X on M, then M is said to be a Kenmotsu manifold (1971). Here ∇ denotes the Levi-Civita connection of g. It is observed that the manifold holds the following relations,

$$\eta(R(X, Y)Z) = \eta(Y)g(X, Z) - \eta(X)g(Y, Z), \qquad (2.3)$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \qquad (2.4)$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \qquad (2.5)$$

$$S(X, \xi) = -(n-1)\eta(X) \qquad (2.6)$$

for all X, Y and Z on M (Kenmotsu (1971), Chaubey & Ojha (2010), Chaubey & R. H. Ojha (2012)). Here R and S denote the Riemannian curvature and Ricci tensors of g, respectively.

A Kenmotsu manifold M is said to be η-Einstein if the non-vanishing Ricci tensor S satisfies the relation $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for all X and Y on M, where a and b are smooth functions on M (Kenmotsu (1971)).

3. NON-SYMMETRIC NON-METRIC CONNECTION

Let M be a Kenmotsu manifold of dimension $(2n+1)$. A linear connection $\tilde{\nabla}$ on M, defined by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(Y)X - g(X, Y)\xi \qquad (3.1)$$

for all vector fields X and Y on M, is known as a non-symmetric non-metric connection (De & Pathak (2004)), if the torsion tensor $\tilde{T}$ of $\tilde{\nabla}$ takes the form $\tilde{T}(X, Y) = \pi(X)Y - \pi(Y)X$ and

$$(\tilde{\nabla}_X g)(Y, Z) = 2\eta(Y)g(X, Z) + 2\eta(Z)g(X, Y). \qquad (3.2)$$

In consequence of the equations (2.1), (2.3) and (3.1), we get

$$\tilde{\nabla}_X \xi = \nabla_X \xi - X - \eta(X)\xi = -2\eta(X)\xi. \qquad (3.3)$$

If $\bar{R}$ denotes the Riemannian curvature tensor with respect to $\tilde{\nabla}$, then it relates to R by the relation

$$\bar{R}(X, Y)Z = R(X, Y)Z - \beta(X, Z)Y + \beta(Y, Z)X - g(Y, Z)(\nabla_X \xi - \eta(X)\xi)$$
$$+ g(Y, Z)(\nabla_Y \xi - \eta(Y)\xi), \qquad (3.4)$$

where β is tensor field of type $(0, 2)$ and defined as

$$\beta(X, Y) = (\nabla_X \eta)(Y) + \eta(X)\eta(Y) + g(X, Y) = 2g(X, Y). \qquad (3.5)$$

In view of (3.5), equation (3.4) takes the form

$$\bar{R}(X, Y)Z = R(X, Y)Z + g(Y, Z)X - g(X, Z)Y + 2[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]\xi. \qquad (3.6)$$

Contracting equation (3.6) along the vector field X, we lead

$$\bar{S}(Y, Z) = S(Y, Z) + 2(n+1)g(Y, Z) - 2\eta(Y)\eta(Z), \qquad (3.7)$$

which gives

$$\bar{Q}Y = QY + 2(n+1)Y - 2\eta(Y)\xi. \qquad (3.8)$$

The contraction of (3.8) gives

$$\bar{\gamma} = \gamma + 2n(2n+3). \qquad (3.9)$$

Here $\bar{\gamma}$ and γ are the scalar curvatures with respect to $\tilde{\nabla}$ and ∇, respectively. $\bar{Q}$ and Q denote the Ricci operators corresponding to the Ricci tensors $\bar{S}$ and S with respect to $\tilde{\nabla}$ and ∇, respectively.

Setting $Z = \xi$ in (3.7) and then using the equations (2.1) and (2.4), we obtain

$$\bar{R}(X, Y)\xi = R(X, Y)\xi + \eta(Y)X - \eta(X)Y + 2[\eta(Y)\eta(X) - \eta(X)\eta(Y)]\xi = 0.$$

This shows that the Kenmotsu manifold M is irregular ($\bar{R}(X, Y)\xi = 0$) with respect to $\tilde{\nabla}$. Thus we state:

Theorem 3.1. *Every $(2n+1)$-dimensional Kenmotsu manifold equipped with $\tilde{\nabla}$ is irregular with respect to $\tilde{\nabla}$.*

4. RICCI SEMI-SYMMETRIC KENMOTSU MANIFOLD WITH A NON-SYMMETRIC NON-METRIC CONNECTION

This section deals with the study of Ricci semi-symmetric Kenmotsu manifold equipped with a non-symmetric non-metric connection $\tilde{\nabla}$. It is well known that

$$(\bar{R}(X,Y)\cdot\bar{S})(Z,U) = -\bar{S}(\bar{R}(X,Y)Z,U) - \bar{S}(Z,\bar{R}(X,Y)U).$$

In view of (2.1) and (3.7), we obtain

$$(\bar{R}(X,Y)\cdot\bar{S})(Z,U) = (R(X,Y)\cdot S)(Z,U) - g(Y,Z)S(X,U) \\ -2(n+1)g(Y,Z)g(X,U) + g(X,Z)S(Y,U) + 2\eta(Y)\eta(Z)g(X,Z)g(Y,U) \\ -2\eta(Y)\eta(U)g(X,Z), \tag{4.1}$$

where $(R(X,Y)\cdot S)(Z,U) = -S(R(X,Y)Z,U) - S(Z,R(X,Y)U)$ for all vector fields X, Y, Z and U on M. If possible, we suppose that $\bar{R}\cdot\bar{S} = R\cdot S$, then (4.1) gives

$$-g(X,Y)S(X,U) - 2(n+1)g(Y,Z)g(X,U) + 2\eta(Y)\eta(Z)g(X,U) \\ -g(Y,U)S(Z,X) + 2g(X,Z)S(Y,U) + 2(n+1)g(X,Z)g(Y,U) \\ -2\eta(Y)\eta(U)g(X,Z) = 0 \tag{4.2}$$

Changing Y and U by ξ in (4.2) and then using the equations (2.1), (2.2), and (2.6), we obtain

$$S(Z,X) = -2ng(Z,X), \quad r = -2n(2n+1), \tag{4.3}$$

which shows that the Kenmotsu manifold endowed with a non -symmetric non-metric connection $\tilde{\nabla}$, under assumption is an Einstein manifold. Also, from equations (3.7) and (4.3), we find

$$\bar{S}(Z,X) = 2g(Y,Z) - 2\eta(Y)\eta(Z). \tag{4.4}$$

This reflects that the Kenmotsu manifold M w.r.t. $\tilde{\nabla}$ under consideration is an η-Einstein manifold. Thus, we can state:

Theorem 4.1. *Let (M,g) be a $(2n+1)$-dimensional Kenmotsu manifold equipped with a non-symmetric non-metric connection $\tilde{\nabla}$. If $\bar{R}\cdot\bar{S} = R\cdot S$ on M, then the manifold to be an Einstein manifold also, M is an η-Einstein with respect to $\tilde{\nabla}$.*

With the help of (4.3), equation (3.9) takes the form $\tilde{r} = 4n$, which shows that if a Kenmotsu manifold equipped with $\tilde{\nabla}$ satisfies $\bar{R}\cdot\bar{S} = R\cdot S$, then the scalar curvature with respect to $\tilde{\nabla}$ is constant. Thus, we state:

Corollary. *If a $(2n+1)$-dimensional Kenmotsu manifold M endowed with $\tilde{\nabla}$ satisfies $\bar{R}\cdot\bar{S} = R\cdot S$, then the scalar curvature of M* with respect to *$\tilde{\nabla}$ to be constant.*

Arslan et al. (2014) proved that

"A semi-Riemannian Einstein manifold M of dimension $n, \geq 4$, satisfies

$$R\cdot C - C\cdot R = \frac{r}{n(n-1)}Q(g,R) = \frac{r}{n(n-1)}Q(g,C), \tag{4.5}$$

where $Q(g,R)$ denotes the *Tachibana tensor* and C is the *conformal curvature tensor*."

From (4.3) and (4.5), we have

$$C\cdot R - R\cdot C = Q(g,R) = Q(g,C). \tag{4.6}$$

Thus, we state:

Corollary. *If a $(2n+1)$-dimensional Kenmotsu manifold M equipped with $\tilde{\nabla}$ satisfies $\bar{R}\cdot\bar{S} = R\cdot S$, then M satisfies the equation (4.6).*

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