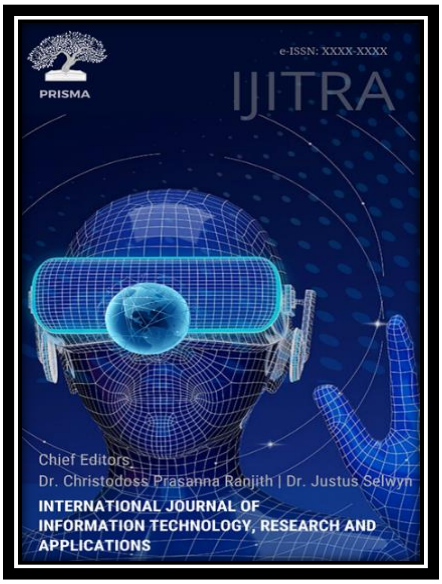

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Characteristics and Operations of Complex Fuzzy Graphs

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ABSTRACT

In this paper, Complex Fuzzy Graph (CFG) analyzed and introduced new concepts in CFG such as Spanning CFG, Complete CFG, path, arc, length, connected, strongest path and weakest path of CFG. We derived some properties of self-complementary CFG and defined the operations on direct product, Semi strong product and strong product of CFG. Derived Isomorphic CFG with example is given in this paper. Moreover, we introduced density of the graph and balanced complex fuzzy graph.

Keywords:

Complex fuzzy graph
Complex fuzzy order relation
Complete CFG
Path-Connected-Strongest path and Weakest path
Direct product
Semi strong product
Strong product
Balanced CFG

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1. INTRODUCTION

Complex Fuzzy Set (CFS) defined by Ramot et al [5], [6] is a new development in the field of fuzzy systems in [12]. Two dimensional membership function defined in complex fuzzy set. Uncertainty and periodicity are involving simultaneously in CFS. So, it paves the way to develop more real life applications in which both uncertainty and periodicity phenomenon involved. The first definition of fuzzy graphs was proposed by Kauffmann in 1973, from the Zadeh's fuzzy relations as in [12, 13, 14].Concepts of complex fuzzy graph, energy of complex fuzzy graph and its applications are derived by Thirunavukarasu et al as defined in [11].

Sunitha [8] introduced and explained about some properties of complete fuzzy graphs. The same technique as in crisp complete graph, is used. Parvathi and Karunambigai [4] extended the concept of fuzzy graphs to introduce intuitionistic fuzzy graphs. In intuitionistic fuzzy graph, both membership and non-membership involved. In 2011, Talal AL-Hawary [9] defined complete fuzzy graph in different manner. Fuzzy order relation, complete fuzzy graph were explained in detailed. From Talal AL-Hawary's [9] and Rosenfeld's [7] perception, we develop a three new operations namely direct product of CFG, semi strong product of CFG and strong product of CFG as in [9]. Thus these concepts will be more helpful to extend the applications of CFG.

The organization of the paper is as follows. Introduction of this paper as described in section 1. In section 2, informative collection of existing concepts of Fuzzy graph and complex fuzzy graph were discussed. Complex fuzzy relation, complex fuzzy sub graph, path, length of the path, strongest and weakest path of

CFG are defined in section 2 . Three new operations of complex fuzzy graph were derived in section 3. Balanced complex fuzzy graph introduced in this paper explained with example in section 4. Further work and concluded of this paper provided in section 5.

2. COMPLEX FUZZY GRAPH

Definition 2.1.

Let V be a nonempty set. A fuzzy graph is a pair of functions G: (ρ, χ) where ρ is a fuzzy subset on V and χ is a symmetric fuzzy relation on σ .

i.e., $\rho : V \to [0,1]$ and $\chi : V \times V \to [0,1]$ such that $\chi(u,v) \le Min(\rho(u), \rho(v))$ for all *u,v* in *V*.

***Definition 2.2.* [12,13]**

A complex fuzzy graph G is a quadruple of the form G = (V, ρ, E, χ), where V is the set of vertices and E $\subseteq VXV$ is a set of edges which are having complex membership function. i.e., $\rho : V \to [0,1]^2$ is a mapping from V to $[0,1]^2$ and $\chi : E \to [0,1]^2$ is a function that maps elements of the form $e \in E : (u,v)$ to $[0,1]^2$, where u $\in V, v \in V$, we assume that V∩E = ϕ . In general, we form the edge $e = (a,b)$ is said to connect the vertices a and b. For an undirected graph both $e_1 = (u,v)$ and $e_2 = (v,u)$ are in the domain of χ .

Example:2.1

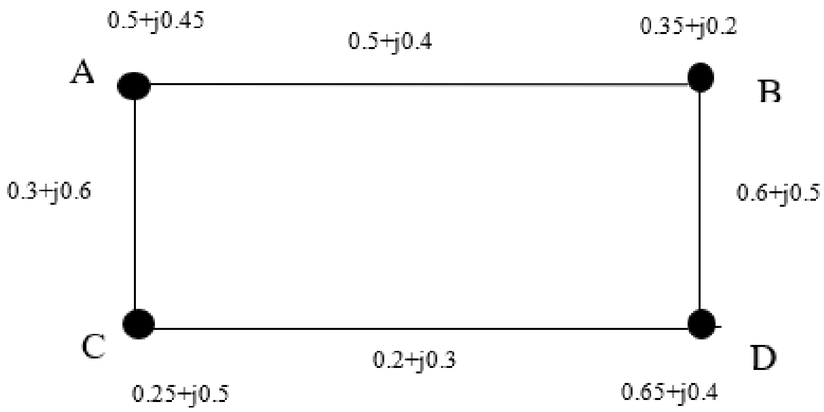

Fig.2.1. Complex Fuzzy Graph $G: (\rho, \chi)$

Definition 2.3

A complex fuzzy graph is a pair $G: (\rho, \chi)$ where $\rho: V \to [0,1]^2$ and $\chi: V \times V \to [0,1]^2$ is a complex fuzzy relation on ρ such that $|\chi(x,y)| \le \min(|\rho(x)|, |\rho(y)|)$ for all $(x,y) \in V$. The underlying crisp graph of G is denoted by $G^*: (\sigma^*, \mu^*)$ where $\sigma^* = \sup(\rho) = \{x \in V, \rho(x) > 0\}. and\ \mu^* = \sup(\chi) = \{(x,y) \in V \times V; \chi(x,y) > 0\}$.

A Complex fuzzy graph $H = (\sigma', \mu')$ is called complex fuzzy sub graph of $G: (\rho, \chi)$,

if $|\sigma'(u)| \le |\rho(u)|$ for all $u \in V$ and $|\mu'(u,v)| \le |\chi(u,v)|$ for all $u, v \in V$. Further H is a spanning complex fuzzy sub graph of G, if $|\sigma'(u)| = |\rho(u)|$ for all $u \in V$.

Example:2.2

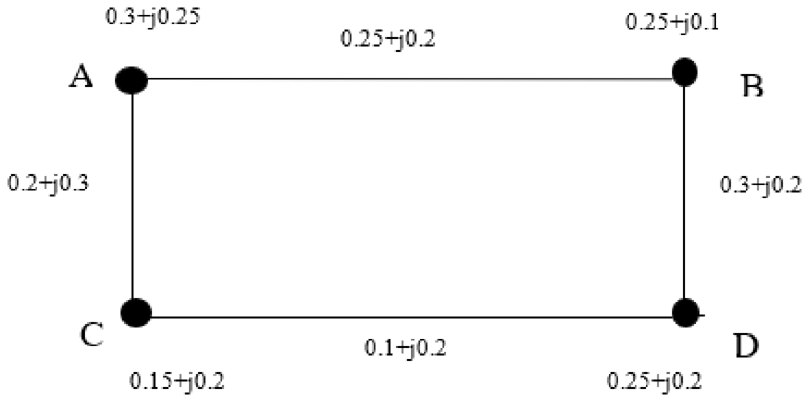

Fig.2.2. Complex Fuzzy sub graph of (Fig.2.1) $G: (\rho, \chi)$

Definition 2.4

A complex fuzzy graph $G: (\rho, \chi)$ is complete if $|\chi(x, y)| = \min(|\rho(x)|, |\rho(y)|)$ for all $(x, y) \in V$.

Example:2.3

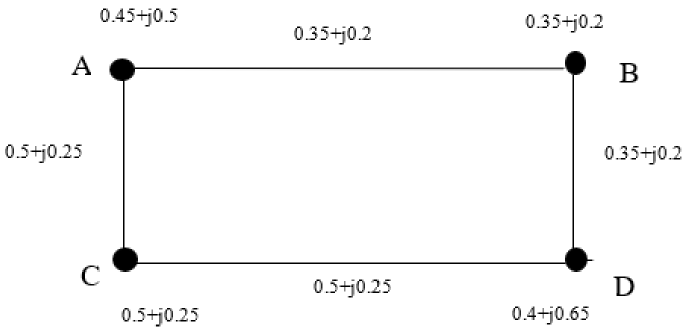

Fig.2.3. Complex Fuzzy Graph $G: (\rho, \chi)$

2.5 Path, Arc, Length of the path, strongest path, weakest path and connected of complex fuzzy graph

A path P in a complex fuzzy graph is a sequence of distinct nodes $u_0, u_1, \ldots, u_n$such that $|\chi(u_{i-1}, u_i)| > 0,\ 1 \leq i \leq n$: where $n \geq\ 0$ is called the length of the path P. The consecutive pairs (u_{i-1}, u_i) are called the arcs of the path. The strength of the path or weight of the weakest arc of the path defined as $Min\{|\chi(u_{i-1}, u_i)|\}_{i=1}^{n}$. The path corresponding to maximum strength is called strongest path and its weight is denoted by $|\chi^{\infty}(u, v)|$. If a path has length zero, then its denoted by $\rho(u_0)$. A path is said to be a cycle if $u_0 = u_n$ and $n \geq 3$. A complex fuzzy graph $G: (\rho, \chi)$ is connected if $|\chi^{\infty}(u, v)| > 0$, for all $u, v \in V$.

Lemma 2.1

Let G:(ρ, χ) be a self-complementary complex fuzzy graph if and only if $\sum_{x,y\in V}|\chi(x, y)| = \frac{1}{2}\sum_{x,y\in V} \min(|\rho(x)|, |\rho(y)|)$.

3. Operations on Complex Fuzzy Graph

Definition 3.1

The direct product of two complex fuzzy graph $G_1:(\rho_1,\chi_1)$ with crisp graph $G_1^* = (V_1,E_1)$ and $G_2:(\rho_2,\chi_2)$ with crisp graph $G_2^* = (V_2,E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$, is defined to be complex fuzzy graph $G_1 \prod G_2 : (\rho_1 \prod \rho_2, \chi_1 \prod \chi_2)$ with crisp graph $G^* = (V_1 \times V_2, E)$, where $E = \{(u_{1,}v_1)(u_{2,}v_2) : (u_{1,}u_2) \in E_1, (v_{1,}v_2) \in E_2\}$

$(\rho_1 \prod \rho_2)(u,v) = Min(|\rho_1(u)|, |\rho_2(v)|)$, for all $(u,v) \in V_1XV_2$ and $(\chi_1 \prod \chi_2)\left((u_{1,}v_1)(u_{2,}v_2)\right) = Min(|\chi_1(u_{1,}u_2)|, |\chi_2(v_{1,}v_2)|)$

Definition 3.2

The semi strong product of two complex fuzzy graph $G_1:(\rho_1,\chi_1)$ with crisp graph $G_1^* = (V_1,E_1)$ and $G_2:(\rho_2,\chi_2)$ with crisp graph $G_2^* = (V_2,E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$, is defined to be complex fuzzy graph $G_1 * G_2 : (\rho_1 * \rho_2, \chi_1 * \chi_2)$ with crisp graph $G^* = (V_1 \times V_2, E)$ where $E = \{(u,v_1)(u,v_2) : u \in V_1, (v_{1,}v_2) \in E_2\} \cup \{(u_{1,}v_1)(u_2,v_2) : (u_{1,}u_2) \in E_1, (v_{1,}v_2) \in E_2\}$,

$(\rho_1 * \rho_2)(u,v) = Min(|\rho_1(u)|, |\rho_2(v)|)$, for all $(u,v) \in V_1XV_2$ and

$(\chi_1 * \chi_2)\left((u,v_1)(u,v_2)\right) = Min(|\rho_1(u)|, |\chi_2(v_{1,}v_2)|)$ and

$(\chi_1 * \chi_2)\left((u_{1,}v_1)(u_2,v_2)\right) = Min(|\chi_1(u_{1,}u_2)|, |\chi_2(v_{1,}v_2)|)$.

Definition 3.3 (Strong Product)

The strong product of two complex fuzzy graph $G_1:(\rho_1,\chi_1)$ with crisp graph $G_1^* = (V_1,E_1)$ and $G_2:(\rho_2,\chi_2)$ with crisp graph $G_2^* = (V_2,E_2)$,where we assume that $V_1 \cap V_2 = \emptyset$, is defined to be complex fuzzy graph $G_1 \otimes G_2 : (\rho_1 \otimes \rho_2, \chi_1 \otimes \chi_2)$ with crisp graph $G^* = (V_1 \times V_2, E)$ where $E = \{(u,v_1)(u,v_2) : u \in V_1, (v_{1,}v_2) \in E_2\} \cup \{(u_1,w)(u_2,w) : w \in V_2, (u_{1,}u_2) \in E_1\} \cup \{(u_{1,}v_1)(u_2,v_2) : (u_{1,}u_2) \in E_1, (v_{1,}v_2) \in E_2\}$

$(\rho_1 \otimes \rho_2)(u,v) = Min(|\rho_1(u)|, |\rho_2(v)|)$, For all $(u,v) \in V_1XV_2$ and

$$(\chi_1 \otimes \chi_2)\left((u,v_1)(u,v_2)\right) = Min(|\rho_1(u)|, |\chi_2(v_{1,}v_2)|),$$
$$(\chi_1 \otimes \chi_2)\left((u_1,w)(u_2,w)\right) = Min(|\rho_2(w)|, |\chi_1(u_{1,}u_2)|),$$
$$(\chi_1 \otimes \chi_2)\left((u_{1,}v_1)(u_2,v_2)\right) = Min(|\chi_1(u_{1,}u_2)|, |\chi_2(v_{1,}v_2)|).$$

Theorem 3.1 Direct product of two complete complex fuzzy graph is also a complete complex fuzzy graph.

Proof

Let $G_1:(\rho_1,\chi_1)$ and $G_2:(\rho_2,\chi_2)$ are two complete complex fuzzy graphs, then

$$(\chi_1 \prod \chi_2)\left((u_{1,}v_1)(u_{2,}v_2)\right) = Min(|\chi_1(u_{1,}u_2)|, |\chi_2(v_{1,}v_2)|)$$
$$= Min(|\rho_1(u_1)|, |\rho_1(u_2)|, |\rho_2(v_1)|, |\rho_2(v_2)|);$$ since G_1 and G_2 are complete.
$$= Min\left((\rho_1 \prod \rho_2)(u_{1,}v_1), (\rho_1 \prod \rho_2)(u_{2,}v_2)\right)$$

Hence, $G_1 \prod G_2$is complete.

Lemma 3.2 Semi strong product of two complete complex fuzzy graph is also a complete complex fuzzy graph.

Lemma 3.3 Strong product of two complete complex fuzzy graph is also a complete complex fuzzy graph.

Theorem 3.2 If G1:(ρ_1, χ_1) and G2 :(ρ_2, χ_2) are complete complex fuzzy graph then

$\overline{G_1 \Pi G_2} \cong \overline{G_1}\ \Pi\ \overline{G_2}$, $\overline{G_1 * G_2} \cong \overline{G_1} * \overline{G_2}and\overline{G_1 \otimes G_2} \cong \overline{G_1} \otimes \overline{G_2}$.

Proof:

Let $\overline{G^*} = (V, \overline{E}), \overline{G} = (\rho, \overline{\chi}) = \overline{G_1 \Pi G_2}, \overline{\chi} = \overline{\chi_1 \Pi \chi_2}, \overline{G_1} = (\rho, \overline{\chi_1}), \overline{G_2} = (\rho, \overline{\chi_2})$, where G is complete complex fuzzy graph and G* is Crisp graph.

Since membership function of edges only considered for conjugate.

So, It is enough to show that $\overline{\chi} = \overline{\chi_1 \Pi \chi_2} = \overline{\chi}_1 \Pi \overline{\chi}_2$

By the property of complex fuzzy set,

$$\overline{(\chi_1 \Pi \chi_2)}\left((u_1,v_1)(u_2,v_2)\right) = 1 - (\chi_1 \Pi \chi_2)\left((u_1,v_1)(u_2,v_2)\right)$$

$$= \left(1 - (\chi_1)\left((u_1,v_1)(u_2,v_2)\right)\right) \Pi \left(1 - (\chi_2)\left((u_1,v_1)(u_2,v_2)\right)\right)$$

$$= \left(\overline{(\chi_1)}\left((u_1,v_1)(u_2,v_2)\right)\right) \Pi \left(\overline{(\chi_2)}\left((u_1,v_1)(u_2,v_2)\right)\right)$$

$$= \overline{(\chi_1} \Pi \overline{\chi_2)}\left((u_1,v_1)(u_2,v_2)\right)$$

Similarly, $\overline{G_1 * G_2} \cong \overline{G_1} * \overline{G_2}and\overline{G_1 \otimes G_2} \cong \overline{G_1} \otimes \overline{G_2}$.

Definition 3.4 Two complex fuzzy graph G1:(ρ_1, χ_1) with crisp graph $G_1^* = (V_1, E_1)$ and G2 :(ρ_2, χ_2) with crisp graph $G_2^* = (V_2, E_2)$ are isomorphic, if there exists a bijection $h: V_1 \to V_2$such that $|\rho_1(x)| = |\rho_2(h(x))|$and $|\chi_1(x,y)| = |\chi_2(h(x), h(y))|$for all $x, y \in V_1$.

Lemma 3.4 Any two isomorphic complex fuzzy graph G1:(ρ_1, χ_1) and G2 :(ρ_2, χ_2) satisfy $\sum_{x \in V_1} |\rho_1(x)| = \sum_{x \in V_2} |\rho_2(x)|$and $\sum_{x,y \in V_1} |\chi_1(x,y)| = \sum_{x,y \in V_2} |\chi_2(x,y)|$.

Definition 3.5 The density of a CFG G:(ρ_1, χ_1) is

$$D(G) = 2\left(\frac{\sum_{u,v \in V} |\chi(u,v)|}{\sum_{u,v \in V} Min(|\rho(u)|, ||\rho(v)||}\right)$$

G is balanced if $D(H) \leq D(G)$ for all complex fuzzy non-empty subgraphs H of G.

Theorem 3.3 Any complete complex fuzzy graph is balanced.

Proof : Let G be a complete complex fuzzy graph. Then

$$D(G) = 2\left(\frac{\sum_{u,v \in V} |\chi(u,v)|}{\sum_{u,v \in V} Min(|\rho(u)|, |\rho(v)|}\right)$$

$$= 2\left(\frac{\sum_{u,v \in V} |\chi(u,v)|}{\sum_{u,v \in V} |\chi(u,v)|}\right), Since\ G\ is\ Complete\ Complex\ fuzzy\ graph$$

$$= 2$$

If H is a non-empty complex fuzzy subgraph of G, then

$$D(H) = 2\left(\frac{\sum_{u,v \in V(H)} |\chi(u,v)|}{\sum_{u,v \in V(H)} Min(|\rho(u)|, |\rho(v)|}\right)$$

$$\leq 2\left(\frac{\sum_{u,v\in V(H)}|\chi(u,v)|}{\sum_{u,v\in V(H)}|\chi(u,v)|}\right)$$

$$= 2\left(\frac{\sum_{u,v\in V}|\chi(u,v)|}{\sum_{u,v\in V}|\chi(u,v)|}\right) = 2 = D(G)$$

Thus G is balanced.

The converse of the preceding result need not true.

4. CONCLUSION

A complex fuzzy set is defined by a complex-valued membership function. Thus, in this way the amplitude term represents uncertainty and the phase term represents periodicity in the uncertainty. Therefore, complex fuzzy graph can handle the redundant nature of uncertainty, incompleteness, indeterminacy, inconsistency, etc. we derived the innovative and important concepts of Complex Fuzzy Graph (CFG) such as complete CFG, Spanning CFG, Path, arc, and weight of CFG are analyzing with its structure. We also defined balanced CFG which is deduced from density of CFG. Its might be effective concepts for finding the coloring, chromatic number, cycles of CFG and its properties.

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