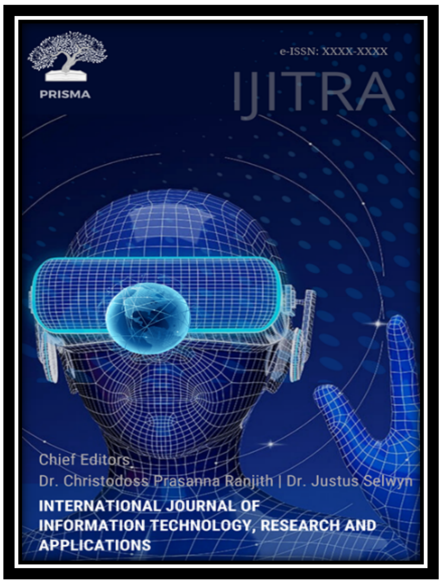


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# Square Distance in Graphs

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**ABSTRACT**

In this paper, we consider a simple connected graph $G = (V(E), E(G))$ having no loops and multiple edges. The order and size of $G$ are denoted by $n$ and $m$ respectively in graphs is a wide branch of graph theory having many scientific and real-life applications. There are various types of distances studied in the literature.The distance $d(u, v)$ is the length of the shortest path between the vertices $u$ and $v$ in $G$. [2]. The detour distance $D(u, v)$ is the length of the longest path between the vertices $u$ and $v$ in $G$. [3]. The concept of centrality widely applied in social network analysis. Degree centrality is the simplest centrality measure to compute centre. Betweenness centrality measures used for analyzed cancer network.



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## I. INTRODUCTION

Graph theory and its applications are vital in many research areas pertaining to connectivity and optimization. Graph Centrality is a prime concept in the field of networking especially applied in problems that orient towards facility location [2]. In this paper, we consider a simple connected graph $G = (V(E), E(G))$ having no loops and multiple edges. The order and size of $G$ are denoted by $n$ and $m$ respectively in graphs is a wide branch of graph theory having many scientific and real-life applications. There are various types of distances studied in the literature.

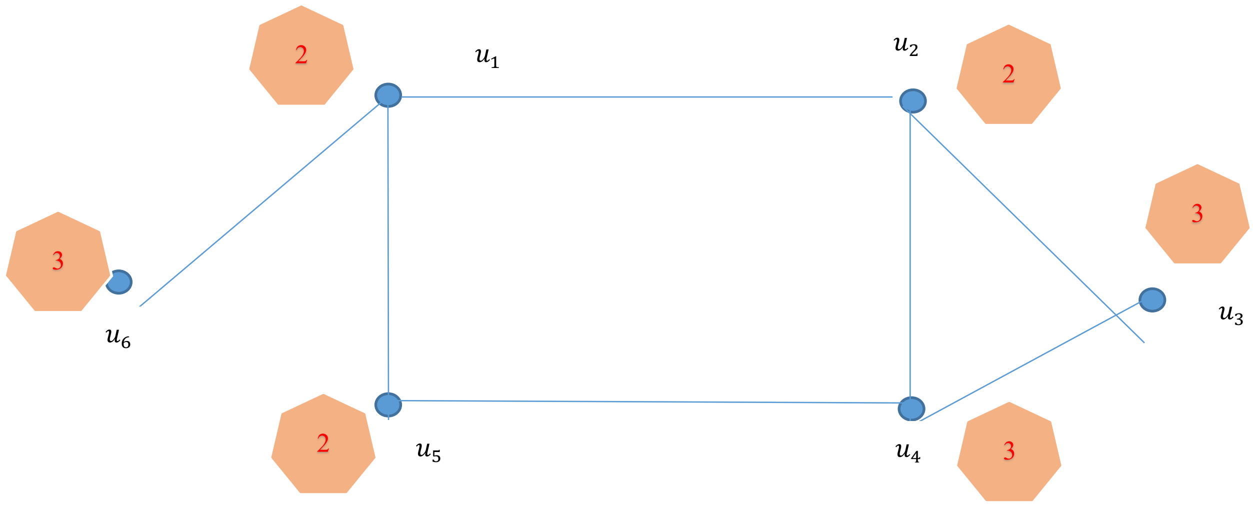
The distance $d(u, v)$ is the length of the shortest path between the vertices $u$ and $v$ in $G$. [2]

The detour distance $D(u, v)$ is the length of the longest path between the vertices $u$ and $v$ in $G$. [3].

The concept of centrality widely applied in social network analysis. Degree centrality is the simplest centrality measure to compute centre. Betweenness centrality measures used for analyzed cancer network. To fix the better centre vertex of the graph is really important for network analysis and cancer networks [4]. The square distance concept is very useful to reduce the number of centers of the given graph.

## II. SQUARE DISTANCE

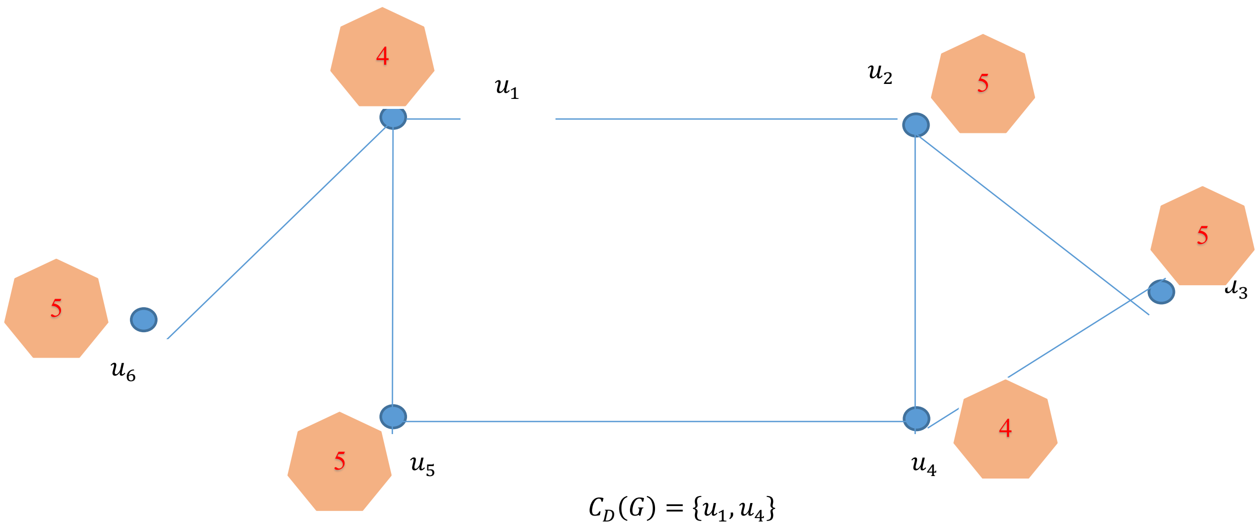
*Definition 2.1:* The square distance of $u$ to $v$ is $[d(u, v) + D(u, v)]^2$ and it is denoted is $Sq\ d(u, v)$. In other words, the square distance of two vertices in a graph $G$ is defined as square of the summation of geodesic and geodetic distance.




$C(G) = \{u_1, u_2, u_5\}$

Geodesic distance of the graph in fig. 1

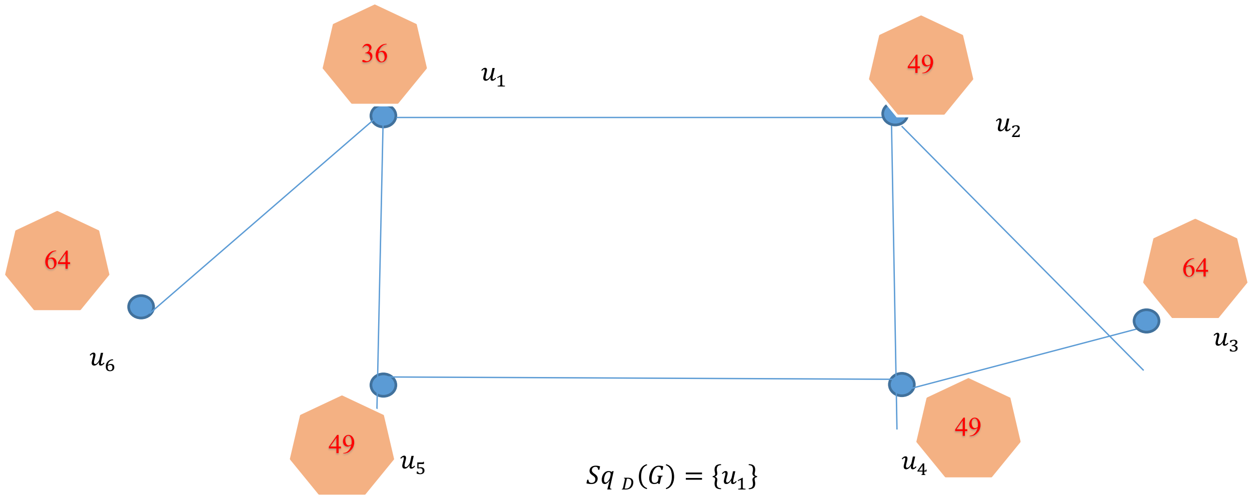
| | | | |
|---|---|---|---|
| $d(u_1, u_2) = 1$ | $d(u_2, u_3) = 1$ | $d(u_3, u_4) = 1$ | $d(u_4, u_5) = 1$ |
| $d(u_1, u_3) = 2$ | $d(u_2, u_4) = 1$ | $d(u_3, u_5) = 2$ | $d(u_4, u_6) = 3$ |
| $d(u_1, u_4) = 2$ | $d(u_2, u_5) = 1$ | $d(u_3, u_6) = 3$ | $d(u_5, u_6) = 2$ |
| $d(u_1, u_5) = 1$ | $d(u_2, u_6) = 2$ | | |
| $d(u_1, u_6) = 1$ | | | |




$C_D(G) = \{u_1, u_4\}$

Geodetic distance of the graph in fig. 2

| | | | |
|---|---|---|---|
| $D(u_1, u_2) = 4$ | $D(u_2, u_3) = 4$ | $D(u_3, u_4) = 4$ | $D(u_4, u_5) = 4$ |
| $D(u_1, u_3) = 4$ | $D(u_2, u_4) = 3$ | $D(u_3, u_5) = 4$ | $D(u_4, u_6) = 4$ |
| $D(u_1, u_4) = 3$ | $D(u_2, u_5) = 3$ | $D(u_3, u_6) = 5$ | $D(u_5, u_6) = 5$ |
| $D(u_1, u_5) = 4$ | $D(u_2, u_6) = 5$ | | |
| $D(u_1, u_6) = 1$ | | | |




Square distance of the graph in fig. 3

The calculation of square distance and square eccentricities of figure 3 has been listed below.

| $Sq\ d(u_i, v_j)$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $Sq\ e(u)$ |
|---|---|---|---|---|---|---|---|
| $u_1$ | 0 | 25 | 36 | 25 | 25 | 4 | 36 |
| $u_2$ | 25 | 0 | 25 | 16 | 16 | 49 | 49 |
| $u_3$ | 36 | 25 | 0 | 25 | 36 | 64 | 64 |
| $u_4$ | 25 | 16 | 25 | 0 | 25 | 49 | 49 |
| $u_5$ | 25 | 16 | 36 | 25 | 0 | 49 | 49 |
| $u_6$ | 4 | 49 | 64 | 49 | 49 | 0 | 64 |

*Definition* 2.2
The square eccentricity of the vertex $u$ is defined as

$$Sq\ e(u) = max\{Sq\ d(u, v)/v \in v(G)\}$$

*Definition* 2.3
The minimum square eccentricity of graph $G$ is called square radius and it is denoted as $Sq\ r(G)$. The maximum square eccentricity of graph $G$ is called square diameter and it is denoted as $Sq\ diam(G)$.

*Definition* 2.4
The set of all minimum square eccentricity vertices are called square centre of graph $G$ and it is denoted as $Sq\ C(G)$.

**Theorem 2.5:**
The square distance is a metric on the set of all vertices of connected graph $G$.

*Proof:*
Let $u, v, w \in V(G)$.
$Sq\ d(u, v) = 0$ if and only if $u = v$, $Sq\ d(u, v) = Sq\ d(v, u)$ for all vertices $u$ and $v$ of $G$.
To prove the triangular inequality,
$Sq\ d(u, w) \leq Sq\ d(u, v) + Sq\ d(v, w)$.
It is true. [since triangular inequality holds for geodesic and geodetic of the graph $G$].

**Theorem 2.6:**

Let $G$ be a graph. Then $Sq\ d(u,v) \geq D(u,v) \geq d(u,v)$

*Proof:*
By the definition, square of the sum of two numbers is bigger than the number.

**Theorem 2.7:**
Let $G$ be a graph. Then $Sq\ r(G) \leq Sq\ diam(G) \leq 2\ Sq\ r(G)$

*Proof:*
The lower bound is true. To prove the upper bound we use triangular inequality.
Let $u, w \in V(G)$, then $Sq\ e(v) = Sq\ r(G)$.
Consider
$$\begin{aligned} Sq\ diam(G) = Sq\ d(u,w) &\leq Sq\ d(u,v) + Sq\ d(v,w) \\ &\leq Sq\ e(v) + Sq\ e(v) \\ &= 2Sq\ e(v) \\ &= 2\ Sq\ r(G). \end{aligned}$$

**Note:**
In figure 1,
$$C(G) = \{u_1, u_2, u_5\}$$
and in figure 2,
$$C_D(G) = \{u_1, u_4\}$$
and in figure 3 $Sq\ C(G) = \{u_1\}$.
Thus the square distance is different from both distance and detour distance for identifying centre of the graph. By applying this concept the number of centers of the given graph has been minimized.

**Result:1**
$C(G) = C_D(G) = Sq\ C(G)$, if and only if $G$ is a tree, path, cycle and complete graph.

## III. SQUARE DISTANCE FOR SOME CLASSES OF GRAPHS:

**Theorem 3.1:**
Let $G$ be a cycle graph $C_n$, $Sq\ r(C_n) = Sq\ diam(C_n) = n^2$, and hence cycle graph is self-centered.

*Proof:*
Let $u_1, u_2, \dots. u_n$ are the vertices of $C_n$. Length of the shortest and longest path of any two vertices are equal in $C_n$.
$d(u_i, u_j) = p$ and $D(u_i, u_j) = m - p$,
For a cycle graph $m = n$
And hence $D(u_i, u_j) = n - p$.
$Sq\ d(u_i, u_j) = \left(d(u_i, u_j) + D(u_i, u_j)\right)^2 = (p + n - p)^2 = n^2$ for every $u_i, u_j$,
$Sq\ e(u_i) = n^2$, theorem holds.

**Theorem 3.2:**
Let $G$ be a path graph $P_n, n \geq 3$.
$$Sq\ r(P_n) = \begin{cases} n^2, n\ is\ even \\ (n-1)^2, n\ is\ odd \end{cases}$$
And $Sq\ diam(P_n) = 4(n-1)^2$.

*Proof:*
Let $G$ be a graph of $n$ vertices of path graph $P_n$. The path between the vertices of $P_n$ is unique. There are $(n-1)$ edges in the graph.
Hence $d(u_i, u_j) = D(u_i, u_j)$
**Case 1: $n$ is even**

| | $u_1$ | $u_2$ | ….. | $u_{\frac{n}{2}}$ | $u_{\frac{n}{2}+1}$ | …… | $u_{n-1}$ | $u_n$ | $Sq\ e(u_i)$ |
|---|---|---|---|---|---|---|---|---|---|
| $u_1$ | 0 | 4 | ….. | $(n-2)^2$ | $n^2$ | …… | $4(n-2)^2$ | $4(n-1)^2$ | $4(n-1)^2$ |
| $u_2$ | 4 | 0 | ….. | $(n-4)^2$ | $(n-2)^2$ | …… | $(n+4)^2$ | $4(n-2)^2$ | $4(n-2)^2$ |
| . . . .<br>$u_{\frac{n}{2}}$ | . . . .<br>$(n-2)^2$ | . . . .<br>$(n-4)^2$ | . . . .<br>….. | . . . .<br>0 | . . . .<br>4 | . . . .<br>…… | . . . .<br>$(n-2)^2$ | . . . .<br>$n^2$ | . . . .<br>$n^2$ |
| $u_{\frac{n}{2}+1}$ | $n^2$ | $(n-2)^2$ | ….. | 4 | 0 | …… | $(n-4)^2$ | $(n-2)^2$ | $n^2$ |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . |
| $u_{n-1}$ | $4(n-2)^2$ | $(n+4)^2$ | ….. | $(n-2)^2$ | $(n-4)^2$ | …… | 0 | 4 | $4(n-2)^2$ |
| $u_n$ | $4(n-1)^2$ | $4(n-2)^2$ | ….. | $n^2$ | $(n-2)^2$ | …… | 4 | 0 | $4(n-1)^2$ |

Hence $Sq\ r(P_n) = n^2$ and $diam(P_n) = 4(n-1)^2$

**Case 2: $n$ is odd**

| | $u_1$ | $u_2$ | … | $u_{\frac{n+1}{2}}$ | $u_{\frac{n+3}{2}}$ | …… | $u_{n-1}$ | $u_n$ | $Sq\ e(u_i)$ |
|---|---|---|---|---|---|---|---|---|---|
| $u_1$ | 0 | 4 | … | $(n-1)^2$ | $(n+1)^2$ | …… | $4(n-2)^2$ | $4(n-1)^2$ | $4(n-1)^2$ |
| $u_2$ | 4 | 0 | … | $(n-3)^2$ | $(n-1)^2$ | …… | $4(n-3)^2$ | $4(n-2)^2$ | $4(n-2)^2$ |
| . . | . . | . . | . . | | . . | . . | . . | . . | . . |
| $u_{\frac{n+1}{2}}$ | $(n-1)^2$ | $(n-3)^2$ | | 0 | 4 | …… | $(n-3)^2$ | $(n-1)^2$ | $(n-1)^2$ |
| $u_{\frac{n+3}{2}}$ | $(n+1)^2$ | $(n-1)^2$ | . . | 4 | 0 | …… | $(n-5)^2$ | $(n-3)^2$ | $(n+1)^2$ |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . |
| $u_{n-1}$ | $4(n-2)^2$ | $4(n-3)^2$ | … | $(n-3)^2$ | $(n-5)^2$ | …… | 0 | 4 | $4(n-2)^2$ |
| $u_n$ | $4(n-1)^2$ | $4(n-2)^2$ | … | $(n-1)^2$ | $(n-3)^2$ | …… | 4 | 0 | $4(n-1)^2$ |

Hence $Sq\ r(P_n) = (n-1)^2$ and $diam(P_n) = 4(n-1)^2$

**Theorem 3.3:**
Let $G$ be a complete graph $K_n$
$Sq\ r(K_n) = Sq\ diam(K_n) = n^2$
*Proof:*
Let $u_1, u_2, \dots, u_n$ be the vertices of $K_n$.
There are two different paths between two vertices of $K_n$, with same vertices. We can consider only one path.
Hence $d(u_i, u_j) = 1, D(u_i, u_j) = n-1$ for all $i = 1,2, \dots n$.
$Sq\ e(u_i) = 1^2 + n^2 - 1^2 = n^2$ for all $i = 1,2, \dots n$.

$$Sq\ r(K_n) = Sq\ diam(K_n) = n^2$$

**Theorem 3.4:**
Let $G$ be a star graph $K_{1,n}$

$$Sq\ r(K_{1,n}) = 4, Sq\ diam(K_{1,n}) = 16$$

*Proof :*
Let $u_1, u_2, \dots, u_n$ be the vertices of $K_{1,n}$.
Let $u_0$ be the adjacent vertices of all other vertices of $K_{1,n}$.
We know that for $K_{1,n}$

$$d(u_0, u_i) = D(u_0, u_i) = 1$$

The square distance between all the vertices are given below.

| | $u_0$ | $u_1$ | $u_2$ | … | $u_n$ | $Sq\ e(u_0, u_n)$ |
|---|---|---|---|---|---|---|
| $u_0$ | 0 | 4 | 4 | … | 4 | 4 |
| $u_1$ | 4 | 0 | 16 | … | 16 | 16 |
| $u_2$ | 4 | 16 | 0 | … | 16 | 16 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $u_n$ | 4 | 16 | 16 | … | 0 | 16 |

Remarks: $C(K_{1,n}) = C_D(K_{1,n}) = SqC(K_{1,n})$

**Theorem 3.5:**
Let G be a cycle graph $C_n$, $Sq\ r(C_n) = Sq\ diam(C_n) = n^2$.

*Proof:*
Let $u_1, u_2, \dots, u_n$ be the vertices of $C_n$. There are $n$ edges in the cycle graph $C_n$.
Hence $d(u_i, u_j) = D(u_i, u_j)$. For each and every pair of vertices, if $d(u_i, u_j) = k$, then $D(u_i, u_j) = n - k$

$$Sq\ d(u_i, u_j) = k^2 + n^2 - k^2 = n^2$$

Hence $Sq\ e(u_i) = n^2$ for all $i = 1,2, \dots n$.

$$Sq\ r(C_n) = Sq\ diam(C_n) = n^2$$

## IV. CONCLUSION:

In graph analytics, centrality is a very important concept in identifying ideal vertex in a graph. Each vertex could be important from an angel depending on how importance is defined. We have various types of centrality measures like Degree centrality, Closeness centrality, Betweenness centrality and Eigen Vector Centrality.
Unweighted networks of bone cell are built and are analyzed using various properties like assortativity, clique, participation, centrality measures and so on.

It is observed that removal of central vertices not impact the topology of the cancer networks, which show that the cancer network cannot be easily disrupted and is resistant to topological changes. The concept of closeness centrality applied to identify the center vertices of networks. Sometimes we need to identify one more minimum number of central vertices of networks. For that the concept of square distance (Result 1) reduce the number of centre vertices of the graph.

In the whole network analysis, central positions always get equated with remarkable leadership, good popularity or excellent reputation. In this paper, we conclude that to identifying central vertices of the graph, with closeness centrality. There are lots of research have done with shortest path and longest path idea. But square distance will reduce the number of center vertices than the geodesic and geodetic studies. The concept we can apply for all unweighted network analysis.

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