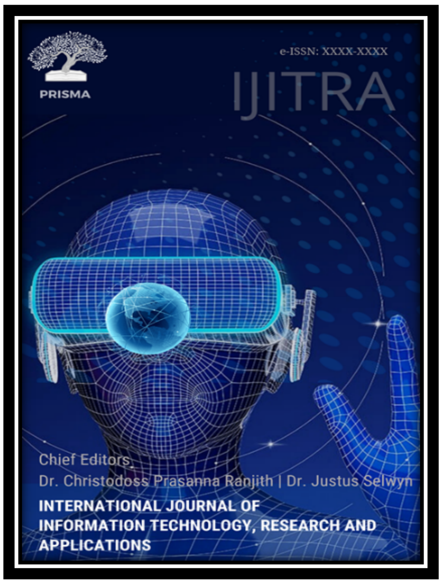

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Inventory model for Exponential Deterioration Items with Power Dependent Demand

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ABSTRACT

In this paper we develop and analyze an inventory model assumption that deterioration rate follows Exponential distributions with power dependent demand. With shortage and without shortage both cases have been taken care of in developing the inventory models. Shortages are fully backlogged whenever they are allowed. Through numerical examples the results are illustrated. The sensitivity analysis for the model has been performed to study the effect changes of the values of the parameters associated with the model.

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I. INTRODUCTION

The influence and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over period of time. In real life, many of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items where deterioration can take place during the normal storage period of the commodity and consequently this loss must be taken into account when analyzing the system. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.

Ghare and Schrader (1963) who developed an economic order quantity model with constant rate of decay. An order-level inventory model for a system with constant rate of deterioration have proposed by Shah and Jaiswal (1977), Aggarwal [1978], Dave and Patel [1981]. Inventory models with a time dependent rate of deterioration were developed by Covert and Philip [1973], Mishra [1973] and Deb and Chaudhuri [1986]. Some of the significant recent work in this area have been done by Chung and Ting [1993], Fujiwara [1993], Hariga [1996], Hariga and Benkherouf[1994], Wee [1995], Jalan et al. [1999], Su, et al. [1996], Chakraborty and Chaudhuri [1997], Giri and Chaudhuri[1997], Chakraborty, et al. [1997] and Jalan and Chaudhuri, [1999],etc.

At the beginning, demand rate were assumed to be constant which is in general likely to be time dependent and stock dependent. Begum et al. [2010] have developed economic lot size model for price-dependent demand. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al [2003], with the motivation of C. K. Tripathy., et al. [2010] and Sushil Kumar., et al. [2013] we developed EOQ models for Weibull deteriorating items and price dependent demand.

In this paper, we have developed generalized EOQ model for deteriorating items where deterioration rate follows two-parameter Weibull and demand rate is considered to be a function of selling price. For the model where shortages are allowed they are completely backlogged. Here we have considered both the case of with shortage and without shortage in developing the model. Using differential equations, the profit rate function are obtained. By maximizing the profit rate function, the optimal production schedule and optimal production quantity are derived. Through numerical illustration the sensitivity analysis is carried. This model also includes some of the earlier models as particular cases for particular or limiting values of the parameters.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions are made for developing the model:

a) The demand rare is a function of selling price which is $f(s) = (a - bs) > 0$
b) Shortages, whenever allowed are completely backlogged.
c) The deterioration rate is proportional to time.
d) Replenishment is instantaneous and lead time is zero.
e) T is the length of the cycle.
f) Q: Ordering quantity in one cycle
g) A: Ordering cost
h) C: Cost per unit
i) h: Inventory holding cost per unit per unit time
j) π: Shortages cost per unit per unit time
k) s: Selling price per unit and
l) The deterioration of units follows the two parameter Weibull distribution with probability density function $f(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}$, $0 < \alpha < 1$ is scale parameter and $\beta > 0$ is shape parameter and $t > 0$. Therefore, the instantaneous rate of replenishment is $\alpha\beta t^{\beta-1}$
m) During time t_1, inventory is depleted due to deterioration and demand of the item. At time t_1 the inventory becomes zero and shortages start occurring.

III. MATHEMATICAL FORMULATION OF THE MODEL

Let I(t) be the inventory level at time 't' ($0 \leq t \leq T$). The differential equations governing the system in the cycle time [0, T] are

$$\frac{d}{dt}I(t) + \alpha\beta t^{\beta-1}I(t) = -(a - bs) \qquad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{d}{dt}I(t) = -(a - bs) \qquad t_1 \leq t \leq T \tag{2}$$

With I(t) = 0 at t = t_1

Solving the equations (1) and (2) and neglecting higher powers of α, we get

$$I(t) = \frac{(a-bs)}{e^{\alpha t^{\beta}}}\left[(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\right] \qquad 0 \leq t \leq t_1 \tag{3}$$

$$I(t) = (a - bs)(t - t_1) \qquad t_1 \leq t \leq T \tag{4}$$

Stock loss due to deterioration in the cycle of length T is

$$L(T) = (a - bs)\int_0^{t_1} e^{\alpha t^{\beta}}dt - (a - bs)\int_0^{t_1} dt$$

$$= (a - bs)\left[\frac{\alpha t_1^{\beta+1}}{\beta+1}\right] \tag{5}$$

Ordering quantity Q in the cycle of length T is

$$Q = L(T) + \int_0^{t}(a - bs)dt$$

$$= (a-bs)\left[\frac{\alpha t_1^{\beta+1}}{\beta+1}\right] + (a-bs)T \tag{6}$$

Holding cost is obtained by substituting the equations (3) and (4), we get

$$H = h\left(\int_0^{t_1} I(t)dt\right) = h\left[\int_0^{t_1}\left[\frac{(a-bs)}{e^{\alpha t^\beta}}\left[(t_1-t)+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})\right]\right]dt\right.$$

Neglecting higher powers of α, we get

$$H = h(a-bs)\left[\left(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1}\right)\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}\right)\right.$$
$$\left. - \left[\int_0^{t_1} te^{-\alpha t^\beta}dt + \frac{\alpha}{\beta+1}\int_{t_1}^{T} t_1^{\beta+1}e^{-\alpha t^\beta}dt\right]\right] \tag{7}$$

Shortage cost during the cycle is

$$S = -\int_{t_1}^{T} I(t)dt = -\int_{t_1}^{T}(a-bs)(t_1-t)dt$$
$$= \frac{1}{2}(a-bs)(T-t_1)^2 \tag{8}$$

Let $P(T, t_1, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have

$$P(T, t_1, s) = s(a-bs) - \frac{1}{T}(A + CQ + H + \pi S) \tag{9}$$

Substituting the values of equations (6), (7) and (8) in equation (9), one can get the profit rate function as

$$P(T, t_1, s) = s(a-bs) - \frac{1}{T}\left[A + C\left[(a-bs)\frac{\alpha t_1^{\beta+1}}{\beta+1} + (a-bs)T\right]\right.$$
$$+h(a-bs)\left[\left(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1}\right)\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}\right) - \left[\int_0^{t_1} te^{-\alpha t^\beta}dt + \frac{\alpha}{\beta+1}\int_{t_1}^{T} t_1^{\beta+1}e^{-\alpha t^\beta}dt\right]\right]$$
$$\left. + \frac{\pi}{2}(a-bs)(T-t_1)^2\right] \tag{10}$$

Let $t_1 = \gamma T$, $0 < \gamma < 1$

Hence we get the profit function

$$P(T, s) = s(a-bs) - \frac{1}{T}\left[A + C(a-bs)\left[\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1} + T\right]\right.$$
$$+h(a-bs)\left[\left((\gamma T)^2 - \left(\frac{\alpha\gamma^{\beta+1}}{\beta+1}\right)^2 T^{2\beta+2}\right) - \left[\int_0^{\gamma T} te^{-\alpha t^\beta}dt + \frac{\alpha}{\beta+1}\int_{\gamma T}^{T}\gamma^{\beta+1}T^{\beta+1}e^{-\alpha t^\beta}dt\right]\right]$$
$$\left. + \frac{\pi}{2}(a-bs)(T-\gamma T)^2\right] \tag{11}$$

Our objective is to maximize the profit function $P(T, s)$. The necessary conditions for maximizing the profit function are

$\frac{\partial P(T,s)}{\partial T} = 0$ and $\frac{\partial P(T,s)}{\partial s} = 0$

We get

$$(a-bs)\left[\frac{C\alpha\beta\gamma^{\beta+1}T^{\beta-1}}{\beta+1} + h\left[\left(\gamma - \frac{\alpha^2\gamma^{2\beta+2}(2\beta+1)T^{2\beta}}{(\beta+1)^2}\right)\right.\right.$$
$$-\left[-\frac{1}{T^2}\left[\int_0^{\gamma T} te^{-\alpha t^\beta}dt + \frac{\alpha}{\beta+1}\int_{\gamma T}^{T}\gamma^{\beta+1}T^{\beta+1}e^{-\alpha t^\beta}dt\right]\right.$$
$$\left.\left.\left. + \frac{1}{T}\frac{\partial}{\partial T}\left[\int_0^{\gamma T} te^{-\alpha t^\beta}dt + \frac{\alpha}{\beta+1}\int_{\gamma T}^{T}\gamma^{\beta+1}T^{\beta+1}e^{-\alpha t^\beta}dt\right]\right] + \frac{\pi}{2}(1-\gamma)^2\right] = 0 \tag{12}$$

and

$$(a-bs)+\frac{b}{T}\left[C\left[\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}+T\right]+h\left[\left((\gamma T)^2-\left(\frac{\alpha\gamma^{\beta+1}}{\beta+1}\right)^2 T^{2\beta+2}\right)\right.\right.$$
$$\left.\left.-\left[\int_0^{\gamma T} te^{-\alpha t^\beta}dt+\frac{\alpha}{\beta+1}\int_{\gamma T}^{T}\gamma^{\beta+1}T^{\beta+1}e^{-\alpha t^\beta}dt\right]\right]+\frac{\pi}{2}(1-\gamma)^2\right]=0 \quad (13)$$

Using the software Mat cad 15, we obtain the optimal policies of the inventory system under study. To find the optimal values of T and s, we obtain the first order partial derivatives of $P(T,s)$ given in equation (11) with respect to T and s and equate them to zero. The condition for maximization of $P(T,s)$ is

$$D=\begin{vmatrix}\frac{\partial^2 P(T,s)}{\partial T^2} & \frac{\partial^2 P(T,s)}{\partial T\partial s}\\ \frac{\partial^2 P(T,s)}{\partial T\partial s} & \frac{\partial^2 P(T,s)}{\partial s^2}\end{vmatrix}<0$$

IV. NUMERICAL EXAMPLE

Case – I (with shortages)

Let A = 500, C = 10, h = 2, π = 0.5, α = 10, β = 0.5, γ = 0.4, a = 100, b = 2

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of $t_1^* = 1.1484$, $T^* = 2.871$, $s^* = 34.976$, $Q^* = 62.627$, $P^*(T,s) = 310.964$

Case – II (without shortages)

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of $t_1^* = 0.8084$, $T^* = 2.021$, $s^* = 18.39$, $Q^* = 120.43$, $P^*(T,s) = 166.199$

V. SENSITIVITY ANALYSIS

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 10%and 20% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in Table-1 and Table-2 for with shortage case and without shortage case respectively. The relationship between the parameters and the optimal values are shown in Figure 1 and 2.

Table – 1

Sensitivity analysis of the model (with shortages)

Variation Parameters	Optimal Policies	Change in parameters				
		-20%	-10%	0%	10%	20%
a	t_1^*	1.285	1.238	1.148	1.020	0.818
	T^*	3.214	3.097	2.871	2.551	2.045
	s^*	31.073	32.883	34.976	38.757	41.65
	Q^*	35.161	52.005	62.627	78.614	138.202
	$P^*(T,s)$	143.527	174.373	310.964	406.978	472.826
b	t_1^*	2.076	1.605	1.148	0.960	0.943
	T^*	5.192	4.014	2.871	2.400	2.359
	s^*	24.868	29.573	34.976	38.314	46.486
	Q^*	99.519	88.286	62.627	40.418	54.076
	$P^*(T,s)$	428.809	411.187	310.964	264.305	172.377
α	t_1^*	0.867	0.867	1.148	1.375	2.067
	T^*	2.168	2.169	2.871	3.438	5.169
	s^*	30.119	30.615	34.976	38.451	40.507
	Q^*	67.11	65.225	62.627	51.286	50.349
	$P^*(T,s)$	350.938	345.726	310.964	215.199	171.88
β	t_1^*	0.804	0.842	1.148	1.686	1.696
	T^*	2.012	2.105	2.871	4.215	4.241
	s^*	30.134	30.168	34.976	37.136	37.152
	Q^*	66.023	65.719	62.627	56.506	50.485
	$P^*(T,s)$	343.059	343.344	310.964	291.625	282.324

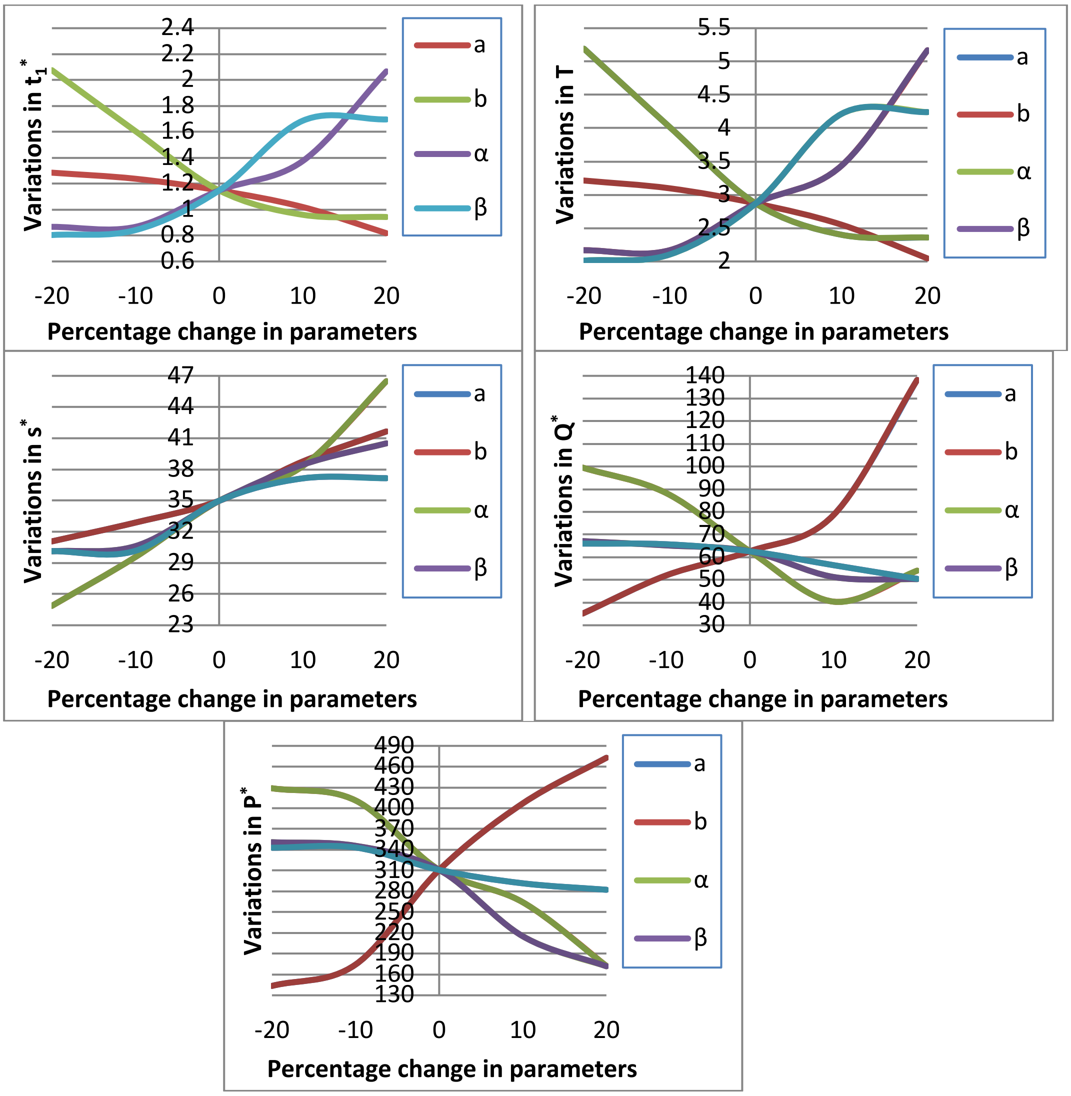


Fig 1: Relationship between parameters and optimal values with shortages

We study from above Table-1 reveals the following

i) Increase in the values of either of the parameters a, will result in increase of T^*, s^* and Q^* but decrease t_1^*, $P^*(T,s)$.

ii) Decrease in the values of either of the parameters a, will result in decrease of T^*, s^* and Q^* but increase t_1^*, $P^*(T,s)$.

iii) Increase in the values of either of the parameters b, will result in increase of s^* but decrease t_1^*, T^*, Q^* and $P^*(T,s)$.

iv) Decrease in the values of either of the parameters b, will result in decrease of t_1^* and s^* but increase T^*, Q^* and $P^*(T,s)$.

v) Increase in the values of either of the parameters α, will result in increase of t_1^* T^* and s^* but decrease Q^* and $P^*(T,s)$.

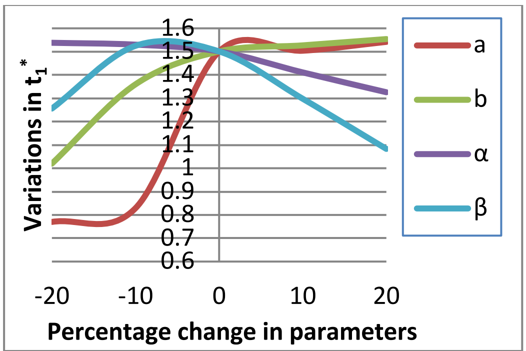
vi) Decrease in the values of either of the parameters α, will result in decrease of t_1^* T^* and s^* but increase Q^* and $P^*(T,s)$.

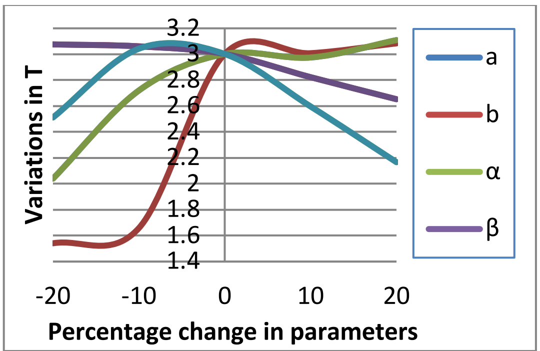
vii) Increase in the values of either of the parameters β, will result in increase of t_1^* T^* and s^* but decrease Q^* and $P^*(T,s)$.

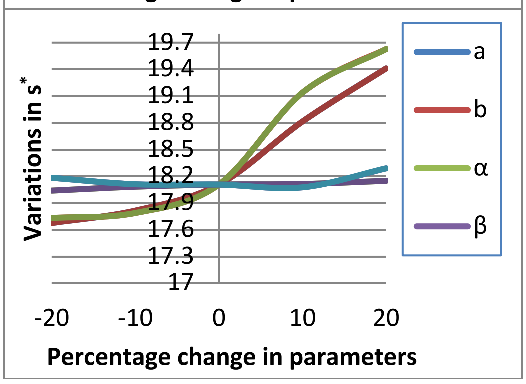
viii) Decrease in the values of either of the parameters β, will result in decrease of t_1^* T^* and s^* but increase Q^* and $P^*(T,s)$.

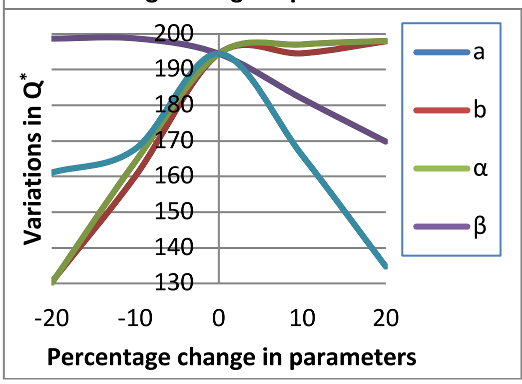
Table – 2
Sensitivity analysis of the model (without shortages)

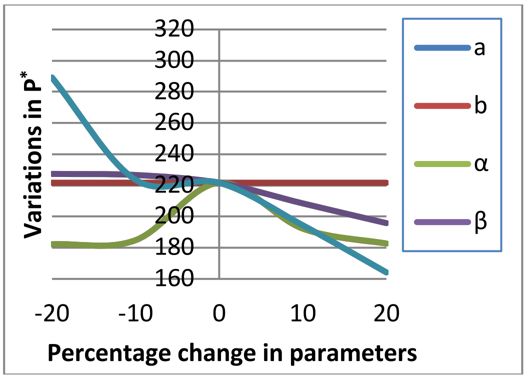
Variation Parameters	Optimal Policies	Change in parameters				
		-20%	-10%	0%	10%	20%
a	t_1^*	0.770	0.829	1.501	1.504	1.542
	T^*	1.541	1.658	3.002	3.008	3.084
	s^*	17.675	17.814	18.107	18.815	19.411
	Q^*	130.339	160.007	194.469	194.575	197.932
	$P^*(T,s)$	171.313	176.242	221.659	261.284	305.507
b	t_1^*	1.02	1.36	1.501	1.487	1.554
	T^*	2.04	2.72	3.002	2.975	3.109
	s^*	17.731	17.788	18.107	19.139	19.628
	Q^*	130.015	164.106	194.469	197.072	198.043
	$P^*(T,s)$	182.2	184.828	221.659	192.267	182.685
α	t_1^*	1.538	1.530	1.501	1.411	1.326
	T^*	3.076	3.061	3.002	2.823	2.653
	s^*	18.039	18.083	18.107	18.113	18.149
	Q^*	198.7	198.76	194.469	181.835	169.794
	$P^*(T,s)$	227.355	226.653	221.659	208.438	195.789
β	t_1^*	1.256	1.525	1.501	1.298	1.083
	T^*	2.512	3.043	3.002	2.597	2.167
	s^*	18.183	18.11	18.107	18.076	18.289
	Q^*	161.089	167.699	194.469	165.891	134.723
	$P^*(T,s)$	289.195	223.586	221.659	194.391	164.166











We study from above Table-2 reveals the following

i) Increase in the values of either of the parameters a, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
ii) Decrease in the values of either of the parameters a, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
iii) Increase in the values of either of the parameters b, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
iv) Decrease in the values of either of the parameters b, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
v) Increase in the values of either of the parameters α, will result in decrease of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
vi) Decrease in the values of either of the parameters α, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
vii) Increase in the values of either of the parameters β, will result in decrease of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.
viii) Decrease in the values of either of the parameters β, will result in increase of t_1^*, T^*, s^*, Q^* and $P^*(T,s)$.

VI. CONCLUSION

In this paper economic production quantity models are developed and analyzed for a single commodity under consideration. It is possible to develop EPQ models for multiple commodities using random production (variable rate of production). Throughout the thesis it is assumed that the money value remain constant over the period of time i.e. the inflation has no influence on the models. It is also possible to develop and analyze the EPQ models developed in this paper with inflation (time values of money) which require further investigation.

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